Applied Mathematics 30

Information Bulletin

2001–2002 School Year

Diploma Examinations Program

Alberta
LEARNING

Contents

Bulletin Information

The purpose of this bulletin is to provide students and teachers of Applied Mathematics 30 with information about the diploma examinations scheduled in the 2001–02 school year.

This bulletin includes descriptions of the Applied Mathematics 30 diploma examinations that will be administered in January, June, and August of 2002, descriptions of the *acceptable standard* and the *standard of excellence*, and subject-specific information. The mark awarded to a student on the Applied Mathematics 30 diploma examinations in 2002 will account for 20% of the student's final blended mark, and the school-awarded mark will account for the remaining 80%.

Teachers are encouraged to share the contents of this bulletin with students.

A Mathematics 33 diploma examination will continue to be available in all regular writing administrations until August 2003. These examinations will be secured (i.e., not released to educators, students, or the general public after the administration). The Mathematics 33 diploma examinations written in 2003 are intended for repeating students, adult students, distance learning students, and students with special circumstances.

For further information regarding program implementation, refer to the Alberta Learning web site at http://www.learning.gov.ab.ca.

Objectives of the Course

The Applied Mathematics 30 course emphasizes the application and relevance of mathematics in daily life. In applied mathematics, numerical and geometrical methods are used to solve problems. When doing investigations or data analysis, algebraic constructs are taught as needed. Tools such as graphing calculators and spreadsheet applications are commonly used to solve problems in applied mathematics.

Students are expected to communicate solutions to problems clearly and effectively when solving both routine and non-routine problems. Technology is to be used for exploration, modelling, and problem solving. Students are also expected to apply mathematical concepts and procedures to real-life problems.

The Applied Mathematics 30 course consists of two sets of outcomes, as specified in the *Program of Studies*. One set of outcomes is common to both the pure and applied mathematics courses, and the second set of outcomes is for the applied mathematics course only.

Standards

Curriculum Standards

Provincial curriculum standards help to communicate how well students need to perform in order to be judged as having achieved the learnings specified for Applied Mathematics 30. These specific statements of standards are written primarily to apprise Applied Mathematics 30 teachers of the extent to which students must know the Applied Mathematics 30 content and must demonstrate the required skills to be able to pass the examination.

Examples of Questions

In this bulletin, there are examples of questions that students should be able to answer in order to demonstrate the *acceptable standard* and the *standard of excellence*. The examples provided are by no means exhaustive; they are intended to provide a profile of acceptable and excellent achievement. Some examples and solutions were developed and validated by classroom teachers of mathematics but have not been validated with students. Other examples were taken from the January 2001 Diploma Examination. All examples model the types of questions and problems that students should be able to solve and the types of activities that students should be able to perform in order to meet the specific outcome to which the questions are linked.

Achievement Standards

Acceptable Standard

Students who attain the *acceptable standard* but not the standard of excellence in Applied Mathematics 30 will receive a final course mark between and including 50% and 79%. Typically, these students have gained new skills and knowledge in mathematics and they can apply mathematical concepts and procedures to find a solution to routine problems. They have demonstrated mathematical skills and knowledge in the six content strands of the Applied Mathematics 30 curriculum and exhibit an ability to apply a broad range of problem-solving skills to these content strands.

Standard of Excellence

Students who attain the *standard of excellence* will receive a final course mark of 80% or higher. Such students have demonstrated their ability and interest in mathematics and have confidence in their mathematical skills. These students can choose the most efficient method for solving problems. They can also find more than one solution and can solve non-routine problems.

Achievement Standards

At a provincial level, it is expected that at least 85% of students will achieve a final course mark of 50% or higher and that at least 15% of students will achieve a final course mark of 85% or higher.

Term Project

Alberta Learning will produce two projects (large-scale mathematical problems) per year for Applied Mathematics 30. These projects are designed to be completed in three to five hours of student time. Use of these projects is optional. Teachers may choose to use the projects as part of their assessment. A sample solution and scoring rubric will be provided for each project. One of the written-response questions worth 10% on the diploma examination will be related to its corresponding project. The first project will be distributed to schools in mid-August 2001, and one of the written-response questions on the January 2002 diploma examination will be related to this project. The second project will be distributed to schools in mid-January 2002, and one written-response question on each of the June 2002 and August 2002 diploma examinations will be related to this project. Students who did not complete the project but who have completed the course will have the knowledge to answer the written-response question; however, students who have completed the project will have had experience with the related mathematical skills in another context.

Examination Specifications

Each Applied Mathematics 30 diploma examination is designed to reflect the core content outlined in the *Applied Mathematics 30 Program of Studies*. The examination is limited to those expectations that can be measured by a paper-and-pencil test. Therefore, the percentage weightings shown below will not necessarily match the percentage of class time devoted to each unit. The examination is developed to be completed in 2.5 h; however, students may take an additional 0.5 h to complete the examination.

The content for the Applied Mathematics 30 diploma examinations in the 2001–02 school year is emphasized as follows.

Machine-Scored Content	*Percentage Emphasis*
Matrices and Pathways	17
Finance and Spreadsheets	17
Statistics and Probabilities	17
Vectors	18
Patterns and Fractals	18
Design	13

Multiple-Choice and Numerical-Response Questions	65
Written-Response Questions	35

Written-response questions assess the degree to which students can draw on their mathematical experiences to solve problems and explain mathematical concepts. Therefore, the written-response questions will not necessarily fall into a particular unit of study but may cover more than one unit or may require students to make the connections between mathematical concepts. Of the seven mathematical processes, the written-response questions will address, communication, connections, reasoning, problem solving, and technology.

Procedural, conceptual, and problem-solving cognitive levels are addressed throughout the examination. The approximate emphasis of each cognitive level is given below.

Multiple Choice, Numerical Response, and Written Response	*Percentage Emphasis*
Procedures	35
Concepts	30
Problem Solving	35

Since the machine-scored and written-response questions are weighted components, a student's examination mark can be calculated using the following equation.

$$\left(\frac{x}{39}\times 65\right)+\left(\frac{y}{10}\times 20\right)+\left(\frac{z}{5}\times 15\right)=\text{exam mark}$$

$x =$ Student's total score on multiple-choice and numerical-response questions
$y =$ Student's total score on the two 10% written-response questions
$z =$ Student's score on the 15% written-response question.

Examination Design

The design of the 2001–02 Applied Mathematics 30 diploma examinations is as follows:

Question Format	*Number of Questions*	*Percentage Emphasis*
Multiple Choice	33	55
Numerical Response	6	10
Written Response	3	35

The six numerical-response questions are interspersed throughout the multiple-choice questions, according to content topic.

The three written-response questions are arranged in the examination in the following manner:

- two questions—each worth 10% of the examination are interspersed throughout the machine-scored questions, according to content topic
- one question—worth 15% of the examination is the last question on the examination

Information required to answer multiple-choice and/or numerical-response questions is often located in a box preceding the question. The number of questions that require the use of the information given in the box will be clearly stated above the box; e.g., "*Use the following information to answer the next two questions.*"

Multiple Choice and Numerical Response

For multiple-choice questions, students are to choose the correct or best possible answer from the four alternatives.

For some numerical-response questions, students are to calculate a numerical answer and to record their answer in a separate area of the answer sheet. When the answer to be recorded cannot be a decimal value, students are asked to determine a whole number value (e.g., the number of people is ______, the degree of this polynomial is ______.) If the answer can be a decimal value, then students are asked to record their answer correct to the nearest tenth or nearest hundredth.

Other numerical-response questions require students to record their understanding of a conceptual idea. The following is an example of such a question.

Use the following information to answer the next question.

$$2\begin{bmatrix} 1 & 0.5 \\ 1.5 & 4 \end{bmatrix} = \begin{bmatrix} a & b \\ c & 8 \end{bmatrix}$$

Numerical Response

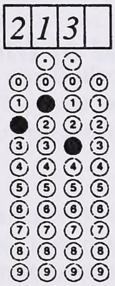
6. In the equation above, the value of

a is ___________ (Record in the **first** column)
b is ___________ (Record in the **second** column)
c is ___________ (Record in the **third** column)

(Record all three digits of your answer in the numerical-response section on the answer sheet.)

Answer: 213

Record 213 on the answer sheet →



Written Response

The written-response questions focus on students' understanding of the process of solving a problem and encourage students to take risks to arrive at a solution. Students will be rewarded for selecting a problem-solving strategy and for carrying through with the strategy to find a solution. The written-response questions focus on the students' use of mathematical processes. To achieve the *standard of excellence*, students must be able to select a strategy, carry it through, and solve the problem correctly. The written-response questions also focus on students' understanding of mathematical concepts and allow for flexibility in evaluating students' communication and problem-solving abilities in mathematics.

Written-response questions ask students to solve, explain, justify, or prove their solution.

In scoring the written-response questions, markers will evaluate how well students

- understand the problem or the mathematical concept
- correctly use the mathematics
- use problem-solving strategies and explain their answer and procedures
- communicate their solutions and mathematical ideas

One of the two 10% questions will be related to the term project provided to all schools by Alberta Learning. The 15% question will focus on the mathematical processes of reasoning and connections.

Above all, students should be encouraged to try to solve all problems. Even an attempt at a solution could be worth some marks. The three written-response questions are each scored with a five-mark scoring rubric. Students should note that markers will be instructed to consider appropriate use of units and appropriate rounding of answers in all solutions to written-response questions. A student will not be penalized more than once for errors of this type.

Assessment of the Mathematical Processes

Open-ended questions provide a way in which to assess mathematics as a common human activity. Open-ended questions allow students to communicate a response by asking them to explain their reasoning, explain their solution, describe mathematical situations, write directions, create new problems, create new strategies, generalize a mathematical situation, and formulate hypotheses.

The written-response questions provide an opportunity to evaluate several of the seven mathematical processes: communication, connections, estimation and mental mathematics, problem solving, reasoning, technology, and visualization. One of the 10% written-response questions will focus on problem solving, and the other 10% question will focus on communication. The 15% written-response question will focus on connections and reasoning. Technology may be incorporated into all three written-response questions.

Communication

"Students need to communicate mathematical ideas clearly and effectively, orally and in writing." (*Applied and Pure Mathematics Program of Studies,* page 6) It is expected that students, in answering the written-response questions, will provide clear, concise answers. Students should clearly communicate all pertinent steps used to arrive at a solution. Students should use correct mathematical notation and conventions. Graphs or diagrams may be presented in solutions.

In communicating an answer, students must be aware of the degree of accuracy required as well as the appropriate units involved.

Problem Solving

"Problem-solving is the focus of mathematics at all grade levels. The development of each student's ability to solve problems is essential. Students develop a true understanding of mathematical concepts and procedures when they solve problems in meaningful contexts." (*Applied and Pure Mathematics Program of Studies,* page 8) Approximately 35% of the machine-scored questions as well as one of the written-response questions will be related to a problem-solving context. It is expected that students will be able to select an appropriate problem-solving strategy to find the solution to a given problem or to model real-world problem situations.

Students should not expect questions on a particular concept to be asked in the same manner every time. They must be able to adapt to changes in the format, which will help to improve their problem-solving skills.

Connections

Students need numerous and varied experiences in order to appreciate the usefulness of mathematics. They must explore connections between different areas of mathematics and between mathematics and other disciplines. The "connections process" also includes relating mathematics to their own daily experiences. "When mathematical ideas are connected to each other through concrete, pictorial, and symbolic representations, students begin to view mathematics as an integrated whole." (*Applied and Pure Mathematics Program of Studies,* page 7)

The connections process is often linked with problem solving and reasoning, as it is an application of these other processes.

Reasoning

"Students need to develop confidence in their ability to reason and to justify their thinking inside and outside of mathematics. The power of reasoning helps students to make sense of mathematics, to make logical inferences from given information, and to convince others of a mathematical meaning.

"Inductive reasoning helps students explore and make conjectures from activities that allow generalizations from a pattern of observations.

"Deductive reasoning helps students test conjectures and build arguments that serve to validate logical thinking. Deductive reasoning also assembles a structured body of knowledge." (*Applied and Pure Mathematics Program of Studies,* page 9)

Reasoning allows students to interpret mathematics and aids in the choice of appropriate problem-solving strategies. Students may be asked to demonstrate logical reasoning when judging the validity of arguments, testing conjectures, and constructing valid arguments.

Technology

"Electronic technologies—calculators and computers—are tools that furnish visual images of mathematical ideas, facilitate organization and analysis of data, and support investigation by students. They allow students to focus on decision making, reflection, reasoning, and problem solving. The computational capacity of technological tools extends the range of problems accessible to students as well as enabling them to execute routine procedures quickly and accurately, thus allowing more time for conceptualizing and modelling. Technology also blurs some of the separations among topics in algebra, geometry, and data analysis by allowing students to connect knowledge and ideas from one area of mathematics to another." *(Principles and Standards for School Mathematics, NCTM 2000,* pages 24 to 25*)*

General Scoring Guide

Scoring Guide for Written-Response Questions

Credit may be given to students who show unusual insight. If their solutions fall outside *specific question scoring rubrics*, they will be scored against the *General Scoring Guide* shown on the next page.

This scoring guide reflects a mark based on four dimensions:

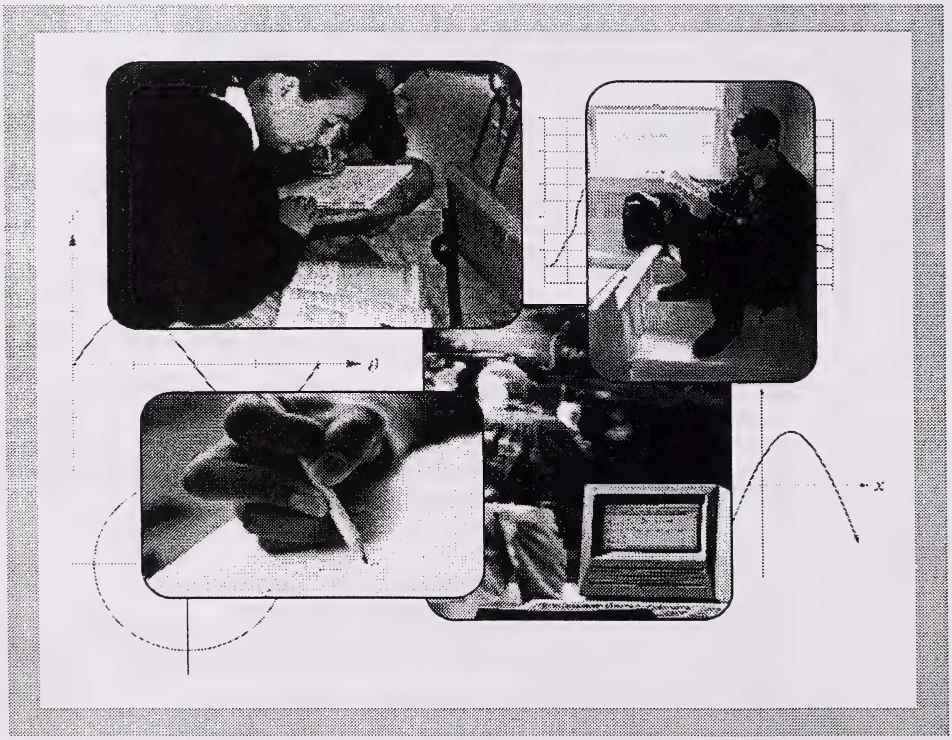
- mathematical understanding
- application of processes
- clarity of communication
- use of technology

GENERAL SCORING GUIDE	
1 mark	In the response, the student • applies some relevant mathematical knowledge to explore the initial stages of the problem; however, the response reflects a misunderstanding of the problem • uses a relevant strategy, mathematical process, or problem-solving technique to explore the initial stages of the problem • communicates very little relevant information and lacks clarity • uses technology inappropriately or the use of technology is not evident
2 marks	In the response, the student • applies some relevant mathematical knowledge to find partial solutions to the problem; however, the response reflects a minimal understanding of the problem • uses relevant strategies, mathematical processes, or problem-solving techniques to find a partial solution to the problem • communicates strategies in a manner that lacks clarity or is incomplete • uses technology where appropriate; however, errors are evident
3 marks	In the response, the student • applies mathematical knowledge to find partial solutions to the problem and reflects a basic understanding of the problem • uses appropriate strategies, mathematical processes, and problem-solving techniques to find partial solutions to the problem • communicates strategies and solutions in an organized manner; however, errors, inconsistencies, and omissions affect clarity • uses technology appropriately; however, there are inconsistencies in their application
4 marks	In the response, the student • applies appropriate mathematical knowledge to find a complete solution to the problem and reflects a good understanding of the problem • uses appropriate strategies, mathematical processes, and problem-solving techniques to find a complete solution to the problem; however, the solution contains an error that hinders understanding of the response • communicates strategies and solutions in an organized manner; however, errors or omissions may affect clarity • uses technology appropriately
5 marks	In the response, the student • applies appropriate mathematical knowledge to find a complete and correct solution to the problem and reflects an excellent understanding of the problem • uses appropriate strategies, mathematical processes, and problem-solving techniques to find a complete, correct solution; the solution may have a minor error but it does not hinder the understanding of the response • communicates strategies and solutions in a clear, complete, and organized manner that reflects a thorough understanding of the problem • uses technology effectively

Applied Mathematics 30

Draft Curriculum Standards and Example Questions

June 2001



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This second draft of the Applied Mathematics 30 Curriculum Standards is intended to assist teachers in preparing for the second year of optional implementation of Applied Mathematics 30. This draft will be revised at the end of the 2001–02 school year, based on input and reactions from the field.

Some of the examples shown in this document were chosen to illustrate the intent of the particular outcomes; however, they will not necessarily be assessed on a diploma examination in the manner shown. These examples were developed and validated by classroom teachers of mathematics but have not been validated with students. Other examples are taken from the Applied Mathematics 30 January 2001 Diploma Examination.

To meet the outcomes of Applied Mathematics 30, students will need access to an approved graphing calculator and a computer with a spreadsheet program. In most classrooms, students will use a graphing calculator on a daily basis. Refer to the Alberta Learning web site www.learning.gov.ab.ca and click *Provincial Testing* for a list of approved graphing calculators.

If you have comments or questions regarding this document, please forward them by e-mail to Shauna Boyce (Shauna.Boyce@gov.ab.ca), or Deanna Shostak, (Deanna.Shostak@gov.ab.ca) phone (780) 427-0010 or fax (780) 422-4454.

The Learner Assessment Branch would like to recognize and thank the many teachers throughout the province who helped prepare this document. We would also like to thank the Curriculum Branch and the Learning Technologies Branch for their input and assistance in reviewing these standards.

Contents

Matrices and Pathways

General Outcomes

Demonstrate an understanding of and proficiency with calculations.

Decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

Describe and apply operations on matrices to solve problems, using technology as required.

Solve problems based on the counting of sets, using techniques such as the Fundamental Counting Principle, permutations, and combinations.

General Notes:
- A node is an intersection of two or more pathways on a grid.
- The intent of the matrices section is that students will recognize situations in which matrices may be applied. This section should not be an in-depth study of matrix operations. Students will need to understand the basic operations and procedures of addition, subtraction, and multiplication of matrices in order to solve problems in a given context.
- Inverse matrices, determinants, and matrix solutions to linear systems are all beyond the scope of Applied Mathematics 30.
- Paper and pencil calculations involving operations on matrices should be limited to those that can be expressed as a matrix of up to size 3 by 3. Technology should be used for matrices of a larger size.

Specific Outcomes

Specific Outcome 1.1

Solve pathway problems, interpreting and applying any constraints. [PS, R]

1.1 Notes:
- A formal study of permutations and combinations is beyond the scope of Applied Mathematics 30.
- The purpose of this outcome is to have students solve pathway problems by recognition and application of a pattern rather than by inspection alone.
- Tree diagrams and other organizational tools (i.e., Pascal's triangle) should be used to present solutions.

Specific Outcome 1.2

Use the Fundamental Counting Principle to determine the number of different ways to perform multistep operations. [PS, R]

1.2 Note:

- Questions should be kept at a level where students who achieve the *Acceptable Standard* can be successful.
- Students should be familiar with the use of factorial notation in order to answer questions involving the Fundamental Counting Principle.

Specific Outcome 1.3

Perform matrix operations of addition, subtraction, and scalar multiplication [CN, PS, R, T, V].

1.3 Note:

- Paper and pencil calculations involving operations on matrices should be limited to those that can be expressed as a matrix of up to size 3 by 3.

Specific Outcome 1.4

Model and solve consumer and network problems, performing matrix multiplication as needed. [CN, PS, T, V]

1.4 Note:

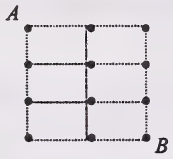
- Problems involving operations on matrices should be limited to those that can be expressed as a matrix of up to size 4 by 4.

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• solve simple pathway problems by inspection or recognition of a pattern • use the Fundamental Counting Principle to determine the number of different ways to perform multistep operations	• recognize and apply a pattern as a solution to a pathway problem • use the Fundamental Counting Principle to determine the number of different ways to perform a series of multistep operations
• build a matrix from a given table	• build a matrix from a given context, and solve problems associated with it
• recognize and produce a row matrix and column matrix • describe the dimensions of a matrix • identify conditions required to perform operations on matrices	
• identify the required matrix operation for a given context • perform operations of addition, subtraction, matrix multiplication, and scalar multiplication on matrices • identify components of a matrix or a matrix operation as they relate to a specific problem	
• make modifications to one or two components of a matrix in response to new scenarios	• make modifications to a matrix in response to new scenarios
• model a simple problem with a matrix • solve problems in which the matrix is given	• model a complex problem with a matrix
• participate in and contribute toward the problem-solving process for problems that require the analysis of matrices and pathways studied in Applied Mathematics 30	• complete the solution to problems that require the analysis of matrices and pathways studied in Applied Mathematics 30

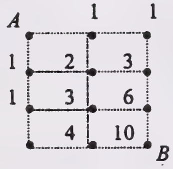
Examples

Students who achieve the *Acceptable Standard* should be able to answer all the following questions, except for any part of a question labelled $\boxed{\text{SE}}$. Parts labelled $\boxed{\text{SE}}$ are appropriate examples for students who achieve the *Standard of Excellence.*

1. Determine all possible paths that lead from point *A* to point *B* if each move must be either down or to the right.

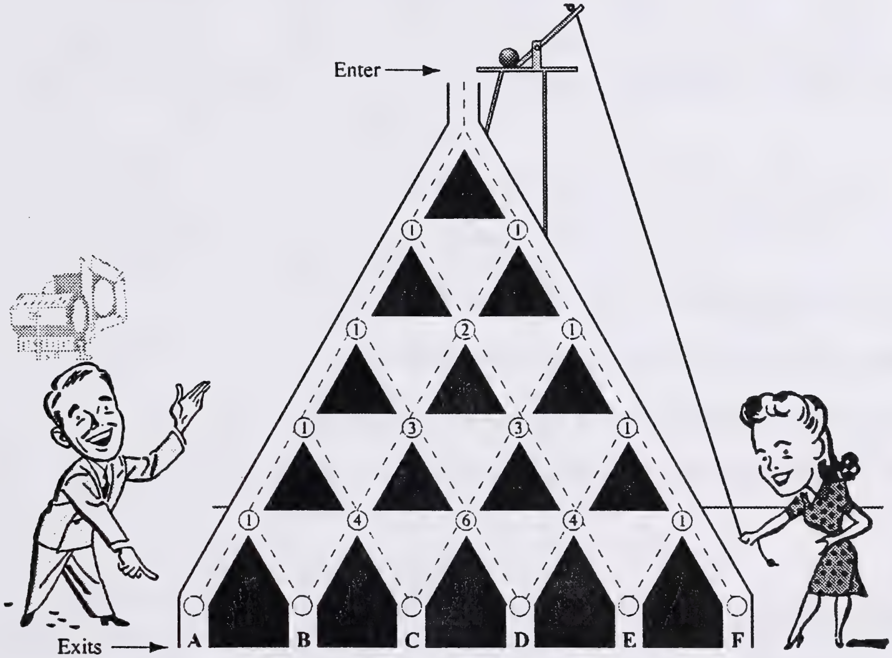


Solution



Number each vertex to show the number of paths that lead to that vertex. A pattern can then be developed that will allow the student to determine the number of paths that lead from point A to point B.

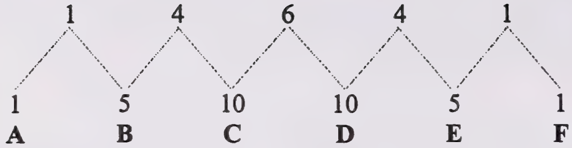
2. To play a particular type of pinball game, a ball is dropped into the top of a machine and moves through the machine until it reaches an exit at the bottom, as illustrated below. The possible paths that the ball can take are indicated by dotted lines, and the numbers at each vertex indicate the number of paths that lead to that vertex.



a. Determine the total number of paths that the ball can take through the machine.
b. For each of the labelled exits, calculate the probability that the ball will exit through it.

Solution

a.



A ball can exit the machine by following a total of $1 + 5 + 10 + 10 + 5 + 1 = 32$ paths.

b. The probability that the ball will exit through exit **A** is $\frac{1}{32}$.

The probability that the ball will exit through exit **B** is $\frac{5}{32}$.

The probability that the ball will exit through exit **C** is $\frac{5}{16}$.

The probability that the ball will exit through exit **D** is $\frac{5}{16}$.

The probability that the ball will exit through exit **E** is $\frac{5}{32}$.

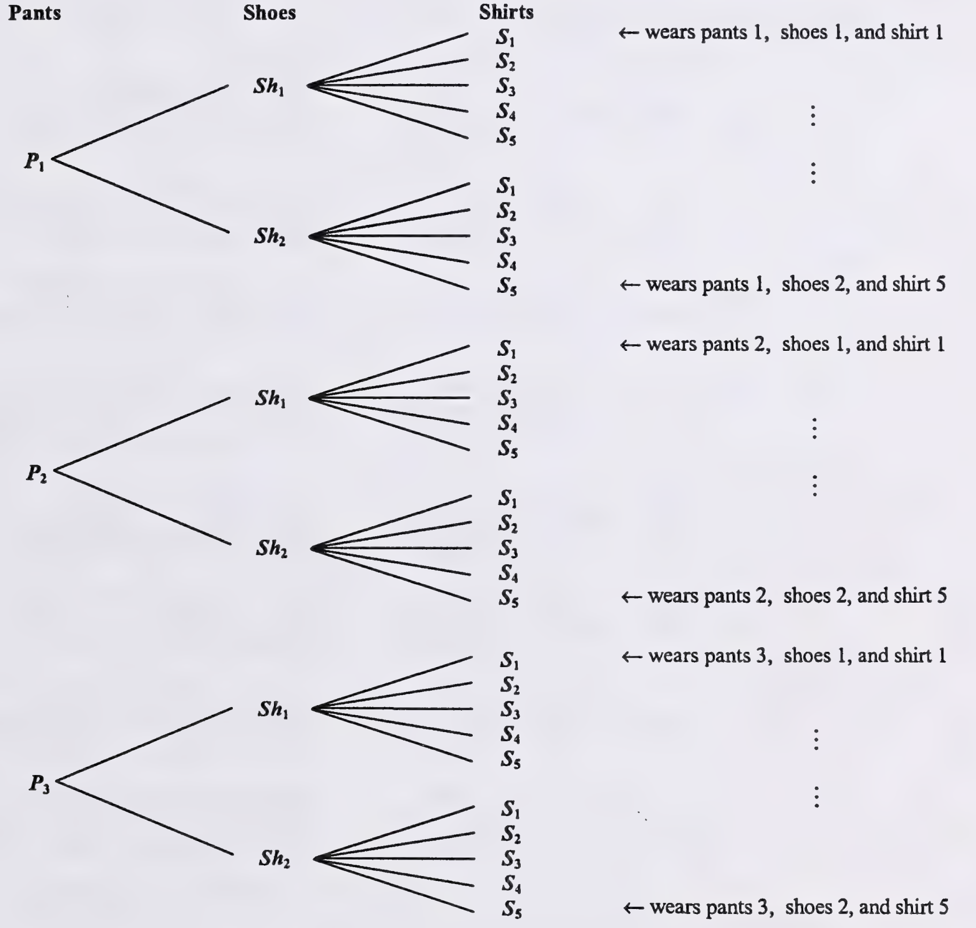
The probability that the ball will exit through exit **F** is $\frac{1}{32}$.

3. Joe has 5 different shirts, 3 different pairs of pants, and 2 different pairs of shoes. An outfit consists of a shirt, a pair of pants, and a pair of shoes. Determine how many possible outfits Joe has by listing all of his possible outfits. Then, use the Fundamental Counting Principle to determine the number of outfits there could be. Do your answers match?

Solution

Method 1: (Sample space)

With his first pair of pants (P_1), Joe can wear one of 2 pairs of shoes and one of 5 shirts. The tree diagram below shows the sample space for this. He can do the same with his second (P_2) and third (P_3) pair of pants, as shown.



30 possible outfits

Method Two: (Fundamental Counting Principle)

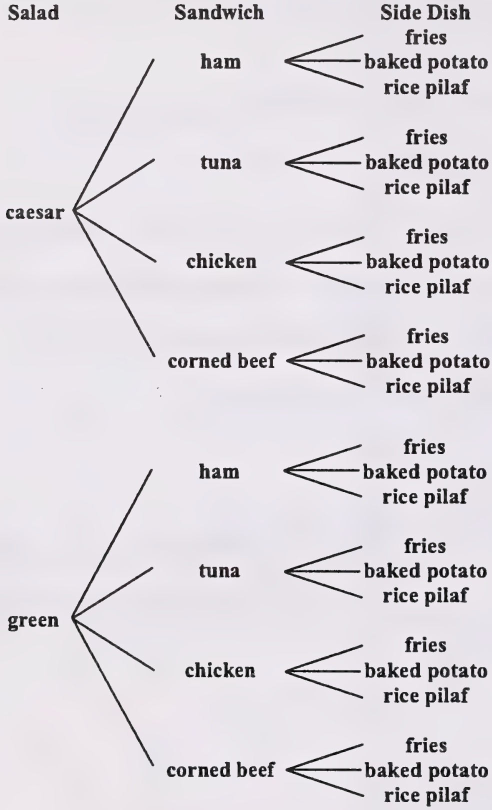
$$\begin{aligned} 3 \text{ pants} \times 2 \text{ shoes} \times 5 \text{ shirts} &= 3 \times 2 \times 5 \\ &= 30 \text{ outfits} \end{aligned}$$

Yes, they match. Both methods show that Joe has 30 possible outfits.

4. A restaurant sells combination plates consisting of 1 salad, 1 sandwich, and 1 side dish. They offer a choice of caesar salad or green salad; ham, tuna, chicken, or corned beef sandwiches; and fries, baked potato, or rice pilaf side dishes.
   a. Use two different methods to determine the number of combinations possible.
   b. Explain how you get the same result with each method.

Solution

a. Method 1: (Sample space)



Combination

- Caesar salad, ham, and fries
- Caesar salad, ham, and baked potato
- Caesar salad, ham, and rice pilaf
- Caesar salad, tuna, and fries
- Caesar salad, tuna, and baked potato
- Caesar salad, tuna, and rice pilaf
- Caesar salad, chicken, and fries
- Caesar salad, chicken, and baked potato
- Caesar salad, chicken, and rice pilaf
- Caesar salad, corned beef, and fries
- Caesar salad, corned beef, and baked potato
- Caesar salad, corned beef, and rice pilaf
- Green salad, ham, and fries
- Green salad, ham, and baked potato
- Green salad, ham, and rice pilaf
- Green salad, tuna, and fries
- Green salad, tuna, and baked potato
- Green salad, tuna, and rice pilaf
- Green salad, chicken, and fries
- Green salad, chicken, and baked potato
- Green salad, chicken, and rice pilaf
- Green salad, corned beef, and fries
- Green salad, corned beef, and baked potato
- Green salad, corned beef, and rice pilaf

24 combinations possible

Method Two: (Fundamental Counting Principle)
There are 2 choices of salad, 4 choices of sandwich, and 3 choices of side dish. Therefore, there are $2 \times 4 \times 3 = 24$ different combinations.

b. Since there are 2 salads, 4 sandwiches, and 3 side dishes to choose from, the Fundamental Counting Principle can be used to determine that there are 24 different combination plates. The sample space simply lists all these combinations.

$\boxed{\text{SE}}$ **5.** How many different arrangements of the letters in the word "MATH" are possible?

Solution 4!

6. Find the matrix B that will solve the following equation.

$$\begin{bmatrix} -5 & 5 \\ 2 & -4 \\ 1 & 6 \end{bmatrix} + [\mathbf{B}] = \begin{bmatrix} -4 & 10 \\ 6 & -7 \\ 3 & 15 \end{bmatrix}$$

Solution

$$[\mathbf{B}] = \begin{bmatrix} -4 & 10 \\ 6 & -7 \\ 3 & 15 \end{bmatrix} - \begin{bmatrix} -5 & 5 \\ 2 & -4 \\ 1 & 6 \end{bmatrix}$$

$$[\mathbf{B}] = \begin{bmatrix} 1 & 5 \\ 4 & -3 \\ 2 & 9 \end{bmatrix}$$

7. Solve the following equation.

$$0.6\begin{bmatrix} -5 & 5 \\ 2 & -4 \\ 1 & 6 \end{bmatrix} + 0.5\begin{bmatrix} -2 & 1 \\ 5 & -7 \\ 3 & 15 \end{bmatrix} = x$$

Solution

$$x = \begin{bmatrix} -4 & 3.5 \\ 3.7 & -5.9 \\ 2.1 & 11.1 \end{bmatrix}$$

8. The following tables show statistics for the Canadian Football League (CFL) Western Division teams during a playing season.

CFL Western Division Statistics

Matrix A

	Against Eastern Teams			
	Win	Tie	Loss	Points
BC	1	0	2	2
Calgary	2	0	0	4
Saskatchewan	1	0	1	2
Edmonton	1	0	2	2

Matrix B

	Against Western Teams			
	Win	Tie	Loss	Points
BC	4	0	0	8
Calgary	3	0	2	6
Saskatchewan	1	0	4	2
Edmonton	1	0	3	2

Use matrix addition to create a single table that combines wins, ties, losses, and points.

Solution

Matrix **A** (against Eastern Teams)

$$\begin{bmatrix} 1 & 0 & 2 & 2 \\ 2 & 0 & 0 & 4 \\ 1 & 0 & 1 & 2 \\ 1 & 0 & 2 & 2 \end{bmatrix}$$

Matrix **B** (against Western Teams)

$$\begin{bmatrix} 4 & 0 & 0 & 8 \\ 3 & 0 & 2 & 6 \\ 1 & 0 & 4 & 2 \\ 1 & 0 & 3 & 2 \end{bmatrix}$$

$$\begin{aligned}[\mathbf{A}] + [\mathbf{B}] &= \begin{bmatrix} 1 & 0 & 2 & 2 \\ 2 & 0 & 0 & 4 \\ 1 & 0 & 1 & 2 \\ 1 & 0 & 2 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 & 8 \\ 3 & 0 & 2 & 6 \\ 1 & 0 & 4 & 2 \\ 1 & 0 & 3 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 5 & 0 & 2 & 10 \\ 5 & 0 & 2 & 10 \\ 2 & 0 & 5 & 4 \\ 2 & 0 & 5 & 4 \end{bmatrix}\end{aligned}$$

The combined table can be shown as

	Win	Tie	Loss	Points
BC	5	0	2	10
Calgary	5	0	2	10
Saskatchewan	2	0	5	4
Edmonton	2	0	5	4

9. An Arctic expedition outfitter offers the following tours. The prices are quoted in Canadian dollars.

Starting Point	Length of Tour and Cost			
	4 days	7 days	10 days	15 days
Yellowknife	$740	$1 145	$1 550	$2 225
Edmonton	$1 640	$2 045	$2 540	$3 125
Toronto	$2 240	$2 645	$3 050	$3 725
Vancouver	$1 940	$2 345	$2 750	$3 425

a. Find a table that compares the Canadian dollar with other currencies.
b. Write a row matrix that represents the cost, in US dollars, of the four tours from a Toronto starting point.
c. Develop a table, in Japanese yen, for the entire price list.

Solution

a. The exchange rates used in the following solution were found at http://www.x-rates.com/tables/CAD.html.

b. $0.6856[2\ 240 \quad 2\ 645 \quad 3\ 050 \quad 3\ 725] = [1\ 535.74 \quad 1\ 813.41 \quad 2\ 091.08 \quad 2\ 553.86]$

c. To convert to Japanese yen, Canadian dollars are multiplied by 74.5629.
To convert the price list to Japanese yen, the following matrix multiplication is used.

$$74.5629\begin{bmatrix} 740 & 1\ 145 & 1\ 550 & 2\ 225 \\ 1\ 640 & 2\ 045 & 2\ 540 & 3\ 125 \\ 2\ 240 & 2\ 645 & 3\ 050 & 3\ 725 \\ 1\ 940 & 2\ 345 & 2\ 750 & 3\ 425 \end{bmatrix}$$

$$= \begin{bmatrix} 55\ 176.55 & 85\ 374.52 & 115\ 572.50 & 165\ 902.45 \\ 122\ 283.16 & 152\ 481.13 & 189\ 389.77 & 233\ 009.06 \\ 167\ 020.90 & 197\ 218.87 & 227\ 416.85 & 277\ 746.80 \\ 144\ 652.03 & 174\ 850.00 & 205\ 047.98 & 255\ 377.93 \end{bmatrix}$$

NB: Solution based on exchange rates as of February 15, 2000.

Therefore, a table that shows the price in Japanese yen is

Starting Point	Length of Tour and Cost			
	4 days	7 days	10 days	15 days
Yellowknife	¥55 176.55	¥85 374.52	¥115 572.50	¥165 902.45
Edmonton	¥122 283.16	¥152 481.13	¥189 389.77	¥233 009.06
Toronto	¥167 020.90	¥197 218.87	¥227 416.85	¥277 746.80
Vancouver	¥144 652.03	¥174 850.00	¥205 047.98	¥255 377.93

10. The matrix below shows the time, in hours, that each of three toys—a car, a truck, and a motorcycle—spend at each of three factory assembly stations, A, B, and C. The "finishing time" for a given toy is the time required for it to pass through all three stations.

$$\begin{array}{r c} & \begin{array}{ccc} \text{A} & \text{B} & \text{C} \end{array} \\ \begin{array}{r} \text{Car} \\ \text{Truck} \\ \text{Motorcycle} \end{array} & \begin{bmatrix} 0.7 & 0.3 & 0.6 \\ 0.6 & 0.4 & 0.5 \\ 0.4 & 0.3 & 0.4 \end{bmatrix} \end{array}$$

a. A toy car, truck, and motorcycle pass through station A. What is the total time that the three toys spend at station A?

b. What is the "finishing time" required for a truck?

c. Changes to the truck's design make it necessary to increase its finishing time by 20%. If the time spent on the truck at each station is increased by the same percentage, what is a matrix operation that will model this change?

Solution

a. The total time spent at station A is the sum of the elements of the first column in the matrix

$$\begin{bmatrix} 0.7 \\ 0.6 \\ 0.4 \end{bmatrix}$$

$0.7 + 0.6 + 0.4 = 1.7$

The total time that the three toys spent at station A is 1.7 hours.

b. The finishing time for a truck is the sum of the elements of the second row of the matrix

$[0.6\ \ 0.4\ \ 0.5]$

$0.6 + 0.4 + 0.5 = 1.5$

A truck's finishing time is 1.5 hours.

c. **Method One:**

Strip out the row that indicates the time the truck requires at each station and make it into a row matrix. This matrix can be multiplied by a scalar of 1.2.

$1.2[0.6\ \ \ 0.4\ \ \ 0.5] = [0.72\ \ \ 0.48\ \ \ 0.6]$

After the design changes, the truck will require 0.72 hours at station A, 0.48 hours at station B, and 0.6 hours at station C.

Method Two:

Perform the following matrix multiplication.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.7 & 0.3 & 0.6 \\ 0.6 & 0.4 & 0.5 \\ 0.4 & 0.3 & 0.4 \end{bmatrix} = \begin{bmatrix} 0.7 & 0.3 & 0.6 \\ 0.72 & 0.48 & 0.6 \\ 0.4 & 0.3 & 0.4 \end{bmatrix}$$

The time that the car and motorcycle spend at each station has not changed, but the time that the truck spends at each station has increased to 0.72 hours at station A, 0.48 hours at station B, and 0.6 hours at station C.

11. Singh's Grocery sells several different kinds of breakfast cereal, each at a different price.
Cereal A is $2.65 per box.
Cereal B is $3.73 per box.
Cereal C is $3.15 per box.
Cereal D is $2.99 per box.

On Wednesday, they sold
5 boxes of cereal A
8 boxes of cereal B
7 boxes of cereal C
10 boxes of cereal D

a. Model the product prices with a row matrix and the product sales with a column matrix.
b. Use matrix multiplication on your matrices to determine Wednesday's revenue from these cereals.
c. Explain why you must use a column and row matrix multiplication to answer this question.

Solution

a. Product price matrix: $\begin{bmatrix} 2.65 & 3.73 & 3.15 & 2.99 \end{bmatrix}$

Sales matrix: $\begin{bmatrix} 5 \\ 8 \\ 7 \\ 10 \end{bmatrix}$

b. Total revenues: $\begin{bmatrix} 2.65 & 3.73 & 3.15 & 2.99 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \\ 7 \\ 10 \end{bmatrix} = \begin{bmatrix} 95.04 \end{bmatrix}$

The revenue from the cereals on Wednesday was $95.04.

c. If you multiply a 1 by 4 matrix by a 4 by 1 matrix, the resulting matrix will be 1 by 1, representing the total revenue. If you try to multiply column matrices or row matrices, the operations are not possible.

12. The administrators of a soccer league are considering changing the way in which points are awarded, especially for the games in which no goals are scored. They want to replace the traditional scheme of 2 points for a win and 1 point for a tie. Three new schemes have been proposed.

Scheme A 3 points for a win
1 point for all ties

Scheme B 3 points for a win
1 point for ties in games that have goals scored
0 points for ties in games with no goals

Scheme C 5 points for a win
3 points for a tie in games with goals scored
0 points for a tie in games with no goals

The statistics for the top four teams in the league after 42 games are shown below.

Team	Wins	Ties with Goals	Ties with no Goals	Losses
Tigers	30	2	8	2
Panthers	24	9	2	7
Pumas	25	7	0	10
Cheetahs	26	1	10	5

a. Using the traditional scheme, determine the points for each team.
b. Using matrix multiplications, model the traditional scheme solution.
c. Using matrices, model and determine the points for each team, using scheme A. Repeat for schemes B and C.
d. Which of the proposed scoring schemes would place the Panthers second in the standings?
e. Which of the proposed scoring schemes would place the Pumas second in the standings?
f. Which of the proposed scoring schemes would place the Cheetahs second in the standings?
SE **g.** Design a scheme that would drop the Tigers out of first place.

Solution

a.

	Wins		Pts		Total Ties		Pts		Total
Tigers	30	×	2	+	10	×	1	=	70
Panthers	24	×	2	+	11	×	1	=	59
Pumas	25	×	2	+	7	×	1	=	57
Cheetahs	26	×	2	+	11	×	1	=	63

b. $$\begin{bmatrix} 30 & 2 & 8 & 2 \\ 24 & 9 & 2 & 7 \\ 25 & 7 & 0 & 10 \\ 26 & 1 & 10 & 5 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 70 \\ 59 \\ 57 \\ 63 \end{bmatrix}$$

Standings

1 Tigers (70 pts)
2 Cheetahs (63 pts)
3 Panthers (59 pts)
4 Pumas (57 pts)

c. Scheme A $$\begin{bmatrix} 30 & 2 & 8 & 2 \\ 24 & 9 & 2 & 7 \\ 25 & 7 & 0 & 10 \\ 26 & 1 & 10 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 100 \\ 83 \\ 82 \\ 89 \end{bmatrix}$$

Standings

1 Tigers (100 pts)
2 Cheetahs (89 pts)
3 Panthers (83 pts)
4 Pumas (82 pts)

Scheme B $$\begin{bmatrix} 30 & 2 & 8 & 2 \\ 24 & 9 & 2 & 7 \\ 25 & 7 & 0 & 10 \\ 26 & 1 & 10 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 92 \\ 81 \\ 82 \\ 79 \end{bmatrix}$$

Standings

1 Tigers (92 pts)
2 Pumas (82 pts)
3 Panthers (81 pts)
4 Cheetahs (79 pts)

Scheme C $$\begin{bmatrix} 30 & 2 & 8 & 2 \\ 24 & 9 & 2 & 7 \\ 25 & 7 & 0 & 10 \\ 26 & 1 & 10 & 5 \end{bmatrix}\begin{bmatrix} 5 \\ 3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 156 \\ 147 \\ 146 \\ 133 \end{bmatrix}$$

Standings

1 Tigers (156 pts)
2 Panthers (147 pts)
3 Pumas (146 pts)
4 Cheetahs (133 pts)

d. Scheme C will place the Panthers second.

e. Scheme B will place the Pumas second.

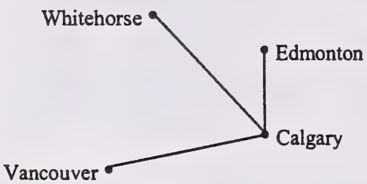
f. Scheme A will place the Cheetahs second.

$\boxed{\text{SE}}$ **g.** Any scheme that awards more points to a win than to a tie will place the Tigers in first. One system that does not do this and, therefore, results in the Tigers falling out of first place is 2 points for a win, 3 points for a tie with goals, and 1 point for a tie with no goals. (This is just one option. Answers may vary.)

13. A particular company's air freight is transported among four cities in Western Canada. Calgary is the hub of the network, which means that all air freight either originates in Calgary or is transported through Calgary.

a. Set up a network matrix, A, for the cities given below. Use 1 to represent that there is a direct freight transport between two cities and 0 to indicate that there is no direct freight transport between two cities.

Cities in Network A



b. If matrix A is squared, matrix A^2 can represent the network matrix for one-way routes and round-trip routes that have exactly one stopover. A stopover occurs when the plane stops in a city enroute to its final destination. Evaluate matrix A^2.

$\boxed{\text{SE}}$ **c.** Explain why there is one element equal to 3, nine elements equal to 1, and six elements equal to 0 in the matrix A^2.

Solution

a. **Matrix A**

$$\begin{array}{c} \\ C \\ E \\ W \\ V \end{array} \begin{array}{c} \begin{array}{cccc} C & E & W & V \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Calgary is the hub of the network and Edmonton, Whitehorse, and Vancouver are all spokes.

b. **Matrix A^2**

$$\begin{array}{c} \\ C \\ E \\ W \\ V \end{array} \begin{array}{c} \begin{array}{cccc} C & E & W & V \end{array} \\ \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

Each entry in A^2 represents the number of one-way routes and round-trip routes that have exactly one stopover.

SE **c.** There is only one element equal to 3 because there are 3 routes with exactly one stopover. You can fly to any one of the three spokes from the Calgary hub and back, with one stopover.

There are nine elements equal to 1 because you can fly from any of the spokes, Edmonton, Whitehorse, or Vancouver, and back with one stopover in Calgary (3 routes), or you can fly between any two of the cities with one stopover (6 routes). This gives a total of 9 routes.

There are six elements equal to 0 because there is no way to fly from the hub to any spoke with exactly one stopover.

14. The matrix below shows the distances of airline routes between Vancouver, Calgary, and Edmonton, with Calgary as the hub.

$$\begin{array}{c c} & \begin{array}{ccc} C & E & V \end{array} \\ \begin{array}{c} C \\ E \\ V \end{array} & \begin{bmatrix} 0 & 300 & 800 \\ 300 & 0 & 0 \\ 800 & 0 & 0 \end{bmatrix} \end{array}$$

Given that Calgary is the hub, then the sum of the distances from the hub to each of the spoke cities and back is (300 + 800 + 300 + 800) or 2 200 km.

a. The matrix below shows the distances of airline routes between Vancouver, Calgary, and Edmonton, with Vancouver as the hub.

$$\begin{array}{c c} & \begin{array}{ccc} V & E & C \end{array} \\ \begin{array}{c} V \\ E \\ C \end{array} & \begin{bmatrix} 0 & 1000 & 800 \\ 1000 & 0 & 0 \\ 800 & 0 & 0 \end{bmatrix} \end{array}$$

Given that Vancouver is the hub, then the sum of the distances from the hub to each of the spoke cities is ____________.

b. The matrix below shows the distances of airline routes between Vancouver, Calgary, and Edmonton, with Edmonton as the hub.

$$\begin{array}{c c} & \begin{array}{ccc} E & V & C \end{array} \\ \begin{array}{c} E \\ V \\ C \end{array} & \begin{bmatrix} 0 & 1000 & 300 \\ 1000 & 0 & 0 \\ 300 & 0 & 0 \end{bmatrix} \end{array}$$

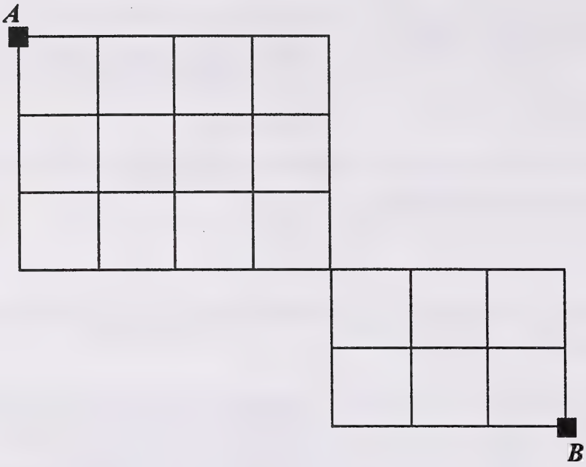
Given that Edmonton is the hub, then the sum of the distances from the hub to each of the spoke cities is ____________.

c. Which is the most efficient hub?

Solution

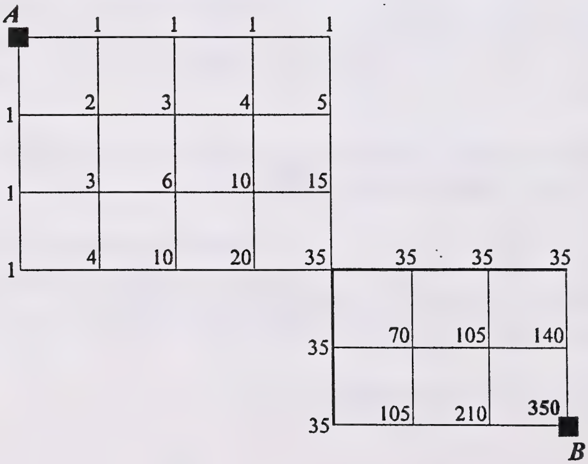
a. (1 000 + 800 + 1 000 + 800) or 3 600 km
b. (1 000 + 300 + 1 000 + 300) or 2 600 km
c. Calgary is the most efficient hub, followed by Edmonton and then Vancouver.

SE 15. Determine the number of "direct" paths that lead from point A to point B on the following grid. A "direct" path must lead either down or to the right.



Solution

Number each vertex to show the number of direct paths that lead to that vertex. The pattern can then be developed that will determine the number of paths that lead from point A to point B.



There are 350 direct paths that lead from point A to point B.

$\boxed{\text{SE}}$ **16.** Solve for x in the following matrix.

$$\begin{bmatrix} 2 & 3 & 5 \\ 4 & 7 & 6 \\ 1 & 2 & 8 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ x & 5 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 33 & 26 \\ 53 & 53 \\ 40 & 21 \end{bmatrix}$$

Solution

Method One: (Algebraic)

$$\begin{aligned} 2(2) + 3(x) + 5(4) &= 33 \\ 4 + 3x + 20 &= 33 \\ 3x &= 9 \\ x &= 3 \end{aligned}$$

Method Two: (Inverse matrix)

$$\begin{bmatrix} 2 & 3 & 5 \\ 4 & 7 & 6 \\ 1 & 2 & 8 \end{bmatrix}^{-1} \times \begin{bmatrix} 33 & 26 \\ 53 & 53 \\ 40 & 21 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 5 \\ 4 & 1 \end{bmatrix}$$

Therefore, the value for x is 3.

Note: Method Two is a recognizable method for solving this equation; however, questions that **require** inverse matrices are beyond the scope of Applied Mathematics 30.

$\boxed{\text{SE}}$ **17.** A washing powder is sold in 6 L and 10 L packages. Market research shows that 40% of the users of the 6 L size switch to the 10 L size for their next purchase and 20% of the users of the 10 L size switch to the 6 L size for the next purchase.

a. If the original market share was 60% for the 6 L size and 40% for the 10 L size, what is the market share for each size in the next round of purchases?

b. If the same percentage of users switch in a third round of purchases, then what will be the market share for each size?

Solution

a. $\begin{bmatrix} 60 & 40 \end{bmatrix} \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix} = \begin{bmatrix} 44 \\ 56 \end{bmatrix}$

The market share is 44% for the 6 L size and 56% for the 10 L size.

b. $\begin{bmatrix} 44 & 56 \end{bmatrix} \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix} = \begin{bmatrix} 37.6 \\ 62.4 \end{bmatrix}$

The market share is 37.6 % for the 6 L size and 62.4% for the 10 L size.

$\boxed{\text{SE}}$ **18.** What conditions must be met in order to perform matrix multiplication? Justify your answer.

Solution

To perform matrix multiplication, the number of columns in the first matrix must match the number of rows in the second matrix.
For example:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \text{ is a 2 by 3 matrix. (2 rows and 3 columns)}$$

and

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \text{ is a 3 by 3 matrix. (3 rows and 3 columns)}$$

When A and B are multiplied, the procedure is as follows:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1(1)+2(4)+3(7) & 1(2)+2(5)+3(8) & 1(3)+2(6)+3(9) \\ 4(1)+5(4)+6(7) & 4(2)+5(5)+6(8) & 4(3)+5(6)+6(9) \end{bmatrix}$$

$$= \begin{bmatrix} 30 & 36 & 42 \\ 66 & 81 & 96 \end{bmatrix}$$

It is impossible to multiply two matrices in which the dimensions do not match, so a product can not be found.
For example:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = [1(1) + 2(3) + 3(\underline{\ ?\ })]$$

The number of columns in the first matrix does not equal the number of rows in the second matrix. Since there is a dimensional mismatch, a product cannot be found.

19. (From Applied Mathematics 30 January 2001 Diploma Examination, Written-Response Question 2)

Use the following information to answer the next question.

The owner of a small amusement park represents the number and type of vehicles that are in his parking lot on a particular Thursday, Friday, and Saturday using the matrix below.

Matrix A: **Day of Week**

$$\begin{array}{c} \\ T \\ F \\ S \end{array} \begin{array}{c} \textbf{Type of Vehicle} \\ \begin{array}{cc} \text{Car} & \text{Bus} \end{array} \\ \begin{bmatrix} 85 & 12 \\ 43 & 17 \\ 102 & 33 \end{bmatrix} \end{array}$$

He makes a second matrix to indicate the parking cost of $8 per car and $22 per bus.

Matrix B: **Type of Vehicle**

$$\begin{array}{c} \\ \text{Car} \\ \text{Bus} \end{array} \begin{array}{c} \textbf{Parking Cost} \\ \begin{bmatrix} 8 \\ 22 \end{bmatrix} \end{array}$$

Written Response—10%

2. **a.** What does the value 33 in matrix A represent?

SOLUTION to part a

There are 33 buses in the parking lot on Saturday.

b. Use matrix multiplication to calculate the revenue for each of the three days. Write a statement that describes the result of this multiplication.

A POSSIBLE SOLUTION to part b

$$\begin{bmatrix} 85 & 12 \\ 43 & 17 \\ 102 & 33 \end{bmatrix} \times \begin{bmatrix} 8 \\ 22 \end{bmatrix} = \begin{bmatrix} 944 \\ 718 \\ 1542 \end{bmatrix}$$

The revenue for each day is \$944, \$718, and \$1 542 for Thursday, Friday, and Saturday respectively.

c. Use matrix operations to calculate an increase of 10% in the daily parking price. Show all calculations.

A POSSIBLE SOLUTION to part c

$$1.10 \times \begin{bmatrix} 8 \\ 22 \end{bmatrix} = \begin{bmatrix} 8.8 \\ 24.2 \end{bmatrix}$$

The new parking price is \$8.80 for cars and \$24.20 for buses.

or

$$0.1 \times \begin{bmatrix} 8 \\ 22 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 2.2 \end{bmatrix}$$

The parking prices will increase by \$0.80 for cars and \$2.20 for buses.

$\boxed{\text{SE}}$ **d.** How much **more** money would the owner have made on Saturday as a result of a 10% price increase?

A POSSIBLE SOLUTION to part d

$$\begin{bmatrix} 85 & 12 \\ 43 & 17 \\ 102 & 33 \end{bmatrix} \times \begin{bmatrix} 8 \\ 22 \end{bmatrix} \times 1.10 = \begin{bmatrix} 85 & 12 \\ 43 & 17 \\ 102 & 33 \end{bmatrix} \times \begin{bmatrix} 8.8 \\ 24.2 \end{bmatrix}$$

$$= \begin{bmatrix} 1038.4 \\ 789.8 \\ 1696.2 \end{bmatrix}$$

or

$$1.1 \times \begin{bmatrix} 944 \\ 718 \\ 1542 \end{bmatrix} = \begin{bmatrix} 1038.4 \\ 789.8 \\ 1696.2 \end{bmatrix}$$

New revenue = \$1 696.20
Orig. revenue = \$1 542.00

\$1 696.20 – \$1 542 = \$154.20

The owner would have made \$154.20 more on Saturday.

Finance and Spreadsheets

General Outcomes

Demonstrate an understanding of and proficiency with calculations.

Decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

Design or use a spreadsheet to make and justify financial decisions.

General Notes:
- Teachers may wish to check with the computer system administrator about log ons for students before beginning this unit.
- This unit is intended to focus on the analysis of financial situations using a spreadsheet, not to be an intense, in-depth study of spreadsheets.
- Students are not expected to master specialized computer functions such as *If-Then-Else* statements.
- Teachers should be aware of time constraints during this unit as some analyses can become quite lengthy. The resource package provides templates of spreadsheets for analytical use.
- Teachers should spend approximately one quarter of the time devoted to this unit in design of the spreadsheet and the remaining time on analyses.
- Students should be familiar with absolute addressing.

Specific Outcomes

Specific Outcome 3.1

Design a financial spreadsheet template to allow users to input their own variables. [C, PS, T]

3.1 Note:
- This outcome is intended to be achieved using computer spreadsheet technology. The tables on the approved graphing calculators do not allow for sufficient manipulation of data. Recursive data can not be entered into these lists.

Specific Outcome 3.2

Analyze the costs and benefits of renting or buying an increasing asset (e.g., home), under different circumstances. [C, CN, PS, T]

3.2 Notes:

- Communicating the results of the analysis is a key idea behind specific outcome 3.2.
- Although calculating mortgage or loan payments using the finance applications of a graphing calculator may be beneficial to classroom discussion, it will not be assessed on the diploma examination. Not all calculators on the approved list have this financial application function.
- Diploma examination questions relating to this outcome will focus on situations where the mortgage payment is known, or where it can be calculated using a given table or chart.
- The graphing calculator assumes a constant interest rate and payment throughout the amortization of the loan. Interesting discussions may result when changes in rate and payment are considered.
- Much of the analysis can be done using an exponential regression model.
- Students are not required to calculate effective interest rate using a formula.

Specific Outcome 3.3

Analyze the costs and benefits of leasing or buying a decreasing asset (e.g., vehicle, computer) under different circumstances. [C, PS, T]

3.3 Notes:

- Communicating the results of the analysis is a key idea behind specific outcome 3.3.
- Limit spreadsheet construction (as this is not the focus of this outcome) to examples where the compounding period matches the payment period.
- The graphing calculator assumes a constant interest rate and payment throughout the amortization of the loan. Interesting discussions may result when changes in rate and payment are considered.
- Much of the analysis can be done using an exponential regression model.

Specific Outcome 3.4

Analyze an investment portfolio by applying concepts such as interest rate, rate of return, and total return. [C, CN, PS, T]

3.4 Notes:

- The intent of this outcome is to discuss and calculate these values with the use of a spreadsheet. The use of the financial applications on some graphing calculators may suffice for classroom instruction.
- Students should be able to use the compound interest formula and recognize it as an exponential function of the form $y = ab^x$ with a b value greater than 1.

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• distinguish the difference between entered values, fixed values, and computed values	
• identify appropriate algebraic functions used in the solution of spreadsheet problems	• interpret a written problem and develop an appropriate spreadsheet
• work within the parameters of an existing spreadsheet to solve non-recursive or recursive problems	• modify an existing spreadsheet to accommodate changing needs and analyze differing factors in each model
• design a simple spreadsheet using formulas and functions to solve non-recursive problems such as billing and design calculations	• design more complex spreadsheets
• design a simple spreadsheet using formulas and functions to solve recursive problems such as loan calculations	
• given a spreadsheet, determine the algebraic formulas used to construct the spreadsheet	
• given two schedules, correctly identify the more appropriate payment model and give appropriate justification for the choice in terms of cost	
• offer a few pros and cons with respect to the chosen payment model	• offer pros and cons that show thorough understanding with respect to the chosen payment model
• use correct terminology to discuss payment options	
• determine whether renting or buying is a more appropriate choice, given specific circumstances	
• discuss the appropriateness of a regression model to a given context	
• recognize the value of an asset as either increasing or decreasing	

• recognize that the value of an increasing asset can be represented by the regression model $y = ab^x$, where $b > 1$	• calculate annual percentage appreciation using exponential regression
• recognize that the value of a decreasing asset can be represented by the regression model $y = ab^x$, where $0 < b < 1$	• calculate annual percentage depreciation using exponential regression
• use an exponential regression model to make predictions	
• offer a few pros and cons with respect to the regression equation	• offer pros and cons that show a thorough understanding with respect to the regression equation
	• set up a spreadsheet that can help in cost and benefit analysis
• describe an investment portfolio using correct terminology	• compare two similar portfolios and make comments with regard to interest rate, rate of return, and total return
• calculate the total return and the average rate of return of an investment	
• participate in and contribute toward the problem-solving process for problems that require the analysis of finance and spreadsheets studied in Applied Mathematics 30	• complete the solution to problems that require the analysis of finance and spreadsheets studied in Applied Mathematics 30

Examples

Students who achieve the *Acceptable Standard* should be able to answer all the following questions, except for any part of a question labelled $\boxed{SE}$. Parts labelled $\boxed{SE}$ are appropriate examples for students who achieve the *Standard of Excellence.*

1. For the following invoice, develop a spreadsheet that calculates the parts and labour costs and requires the operator to input a minimum number of entries.

Acme Auto Parts

Customer Inquiries

Item No.	Auto Parts	Quantity	Unit Price	Total	Labour	
1	Brake Pads	1	$26.34	$26.34	OH/Front Brakes	$47.70
					(0.9 hrs. @ $53.00/hr.)	
2	Wheel Seals	2	$5.25	$10.50	Machined and	$10.00
					Replaced Rotor	
					(flat rate)	
3	Rotor	1	$30.16	$30.16		
	Total Parts			$67.00	Total Labour	$57.70
					Subtotal	$124.70
					GST (7%)	$8.73
					Total	$133.43

Solution

	A	B	C	D	E	F	G
1			Acme Auto Parts				
2	Customer Inquiries						
3	Item No.	Auto Parts	Quantity	Unit Price		Labour	
4	1	Brake Pads	1	$26.34	=C4*D4	OH/Front Brakes	$47.70
5						(0.9 hrs. @ $53.00/hr.)	
6	2	Wheel Seals	2	$5.25	=C6*D6	Machined and	$10.00
7						Replaced Rotor	
8						(flat rate)	
9	3	Rotor	1	$30.16	=C9*D9		
10							
11				Total Parts	=SUM(E4:E10)	Total Labour	=SUM(G4:G10)
12						Subtotal	=E11+G11
13						GST (7%)	=(G12)*0.07
14						Total	=G12+G13

2. A person purchases a car for $25 000. The value of the car changes over time, as shown in the chart below.

Year	Value ($)
0	25 000
1	17 500
2	14 000
3	12 600
4	11 340
5	10 206

a. Is this car an increasing or decreasing asset? Support your answer.
b. Perform an exponential regression on the data and state the regression equation.
c. Based on the regression model, predict the value of the car in year 10.
SE **d.** What is the average annual percentage depreciation on this car?

Solution

a. The car can be identified as a decreasing asset because its value decreases over time.
or
The car can be identified as a decreasing asset because the regression model in the form $y = ab^x$ results in a value for b that is less than 1 and greater than 0.
b. The regression equation is $y = 21\ 932.84(0.845)^x$.
c. In year 10, the car will be worth $4 080.18.
SE **d.** Since the value of b in $y = ab^x$ is 0.845, the average annual depreciation is $1 - 0.845 = 0.155$, or 15.5% per year.

3. An individual invested \$1 000 into a GIC at his local financial institution 10 years ago. During the last ten years, he has been tracking his investment by recording the value of his GIC at the end of each year. The data are shown below.

Year	Value of GIC
1	\$1 052.00
2	\$1 101.97
3	\$1 156.19
4	\$1 226.14
5	\$1 298.11
6	\$1 368.60
7	\$1 449.62
8	\$1 544.57
9	\$1 638.02
10	\$1 754.32

Using your graphing calculator, enter these values into a table and perform an exponential regression on the data. Use the regression model to answer the following questions.

a. Predict the value of this GIC in 20 years. Predict its value in 30 years.

b. Discuss whether you feel that the predictions made in part a would be accurate. What fundamental assumptions must be taken into consideration when making these predictions?

SE **c.** What is the average annual rate of return on this investment?

SE **d.** Would any of the given years' rates, as calculated from the above chart, necessarily have the same value as the answer given in part c? Explain.

Solution

The exponential regression model for this data is $y = 980.815...(1.058...)^x$.

a. Value of GIC in 20 years

$y = 980.815...(1.058...)^{20}$
$y = \$3\,057.67$

The value of the GIC after 20 years is predicted to be \$3 057.67.

Value of GIC in 30 years

$y = 980.815...(1.058...)^{30}$
$y = \$5\,398.74$

The value of the GIC after 30 years is predicted to be \$5 398.74.

b. A basic assumption that is being made is that there is little or no major increase to the rate of return over the next 20 to 30 years. The predictions in part a would probably not be very accurate since the rate of return seems to be rising over time, and therefore, it is reasonable to expect the rate to continue to increase. This would have the final result of increasing the average annual return over time and, hence, increasing the value of the GIC.

SE **c.** The increasing asset equation is $y = 980.82(1.058)^x$; therefore, the average annual rate of return is 5.8%/a..

SE **d.** No. The overall change is calculated at 5.8%/a, but each of the individual years could be greater or less than that value without ever being equal to 5.8%/a.

4. A person has recently received $10 000 and is trying to decide what to do with it. He currently has a $70 000 mortgage that charges 8.35%/a, compounded semi-annually, and that is coming due this month. The mortgage is currently ammortized over 15 years. He could put the money into an investment that pays 7.9%/a, compounded monthly, or he could put the money toward the mortgage.

The current mortgage payments are $677.40/mo, and after 15 years, the total interest paid will be $51 932.

The following information shows the details of the options he is considering.

Option One: Invest the money into an annuity.

If the $10 000 is invested at 7.9%/a, compounded monthly, in 15 years it will be worth $32 580.09.

Option Two: Put the money toward the mortgage, and invest the difference in mortgage payments into an annuity.

If he puts the $10 000 toward his mortgage, the new monthly payments will be $580.63/mo, and after 15 years, the total interest paid will be $44 513.40.

If the difference in payments ($96.77) is invested, the value of the annuity in 15 years will be $33 191.02.

Option Three: Put all the money toward the mortgage, but leave the monthly payments the same, and, at the end of the ammortization period, invest an amount equivalent to the mortgage payments for the remainder of the 15 years.

If he puts the $10 000 toward the mortgage, he can pay off his house in 11 years and 5 months instead of 15 years. The total cost to mortgage the home under this option will be $32 520.66.

If, at the end of the amortization period, an amount equivalent to the mortgage payments ($677.40) is invested for the remainder of the ammortization period, the value of the investment in 3 years 7 months will be $33 542.62.

a. Determine the net cost of each option by determining the interest paid on the mortgage and the interest earned from the investment.
b. Analyze the different options to determine which option you would recommend and explain why.
c. Identify a fourth option to consider, and calculate the net cost of this option. You may wish to use the financial applications on your graphing calculator.

Solution

a. **Option One:**

	Interest paid on mortgage	$51 932.00
	Interest earned from investment	$22 580.09
	Net Cost	$29 351.91

Option Two:

	Interest paid on mortgage	$44 513.40
	Interest earned from investment	$15 772.42
	Net Cost	$28 739.58

Option Three:

	Interest paid on mortgage	$32 520.66
	Interest earned from investment	$4 414.42
	Net Cost	$28 106.24

b. The third option costs the least amount of money over the 15 years (by $1 245.67 over option one, and by $633.34 over option two). If this were the only criterion, option three should be recommended. However, it may be difficult for the person to invest this amount of money immediately after the mortgage is paid off, and therefore, it may be easier to put it toward the mortgage, and invest the difference into an annuity every month (option two).

c. A fourth option could be to split the money between the mortgage and the investment, and also invest the difference in mortgage payments.

   If the person put $5 000 toward the mortgage and invested the other $5 000 and the difference in mortgage payments every month, the payments on the $65 000 mortgage would be $629. 02. The interest paid on the mortgage would be $48 223.60.

   If the $5 000 is invested into the annuity, in 15 years, it will be worth $16 290.05. This means that the interest earned is $11 290.05.

   If the difference in the mortgage payment ($48.38) is invested every month into the annuity as well, the value of this portion will be worth $16 593.80. This means that the interest earned will be $7 885.40.

   The net cost of this option is $29 048.15. (This is just one example of a fourth option. Answers may vary.)

5. The Smith family has decided to take out a loan to purchase a new recreational vehicle. Their bank has offered them a loan $30 000 over a period of 10 years with an interest rate of 6.25%/a compounded monthly. The dealership selling the RV has offered them a loan of $30 000 over a period of 12 years with an interest rate of 6.00%/a compounded monthly. The Smiths' set up a spreadsheet for each option. Portions of both spreadsheets are displayed below.

Option 1

Loan	$30,000.00
Interest rate	6.25%/a comp. monthly
Period	120 months
Payment	$336.84

Month	Balance	Interest	Payment	New Balance
1	$30,000.00	$156.25	$336.84	$29,819.41
2	$29,819.41	$155.31	$336.84	$29,637.88
3	$29,637.88	$154.36	$336.84	$29,455.40
4	$29,455.40	$153.41	$336.84	$29,271.98
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
117	$1,330.04	$6.93	$336.84	$1,000.13
118	$1,000.13	$5.21	$336.84	$668.50
119	$668.50	$3.48	$336.84	$335.14
120	$335.14	$1.75	$336.84	$0.05
Totals		**$10,420.85**	**$40,420.80**	

Option 2

Loan	$30,000.00
Interest rate	6.00% /a comp. monthly
Period	144 months
Payment	$292.76

Month	Balance	Interest	Payment	New Balance
1	$30,000.00	$150.00	$292.76	$29,857.24
2	$29,857.24	$149.29	$292.76	$29,713.77
3	$29,713.77	$148.57	$292.76	$29,569.58
4	$29,569.58	$147.85	$292.76	$29,424.66
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
141	$1,155.53	$5.78	$292.76	$868.55
142	$868.55	$4.34	$292.76	$580.13
143	$580.13	$2.90	$292.76	$290.27
144	$290.27	$1.45	$292.76	-$1.04
Totals		**$12,156.40**	**$42,157.44**	

a. What is the difference in the monthly payments between the two options?
b. What is the difference in the total interest payments between the two options?
c. Explain the differences between option 1 and option 2. Does a lower interest rate necessarily mean paying less interest? Explain.

SE d. At the end of the payment period for option 1, there is a balance of \$0.05 and at the end of the payment period for option 2 there is a balance of –\$1.04. Explain how the financial institution would handle these values.

Solution

a.

Option 1	\$336.84
Option 2	\$292.76
Difference	\$ 44.08

b.

Option 2	\$12 156.40
Option 1	\$10 420.85
Difference	\$ 1 735.55

c. Option 1 has a higher interest rate but a shorter time period. Because of the longer period, a lower interest rate doesn't necessarily amount to less interest, since you pay for a longer period.

SE d. In option 1, the balance of \$0.05 is still owing and, therefore, it would be added to the last payment to make the last payment equal \$336.89. In option 2, the balance of –\$1.04 would be an overpayment and, therefore, the last payment would be decreased to \$291.72.

6. Chris is looking into the purchase/lease of a car. The vehicle he has choosen is priced at \$21 720.00 plus GST, and financing is calculated at 7.5%/a, compounded monthly. Chris will make a down payment of \$2 000 and has the following two options available.

Option 1: (Lease with buyout)

Amount financed	\$19 720
Monthly lease payments	$\$377.55 + \text{GST on monthly payment} + \dfrac{\text{GST on down payment}}{36} = 407.87$
Number of months	36
Final buyout	\$9 394.06 + GST = \$10 051. 64

Option 2: (Finance full amount)

Purchase price	\$21 720 + GST = \$23 240.40
Amount financed	\$23 240.40 – 2000.00 = \$21 240.40
Monthly payment	\$660.71
Number of months	36

Note: When leasing a car, the GST is calculated on the monthly payment. The GST on the down payment is divided by the number of months and also added to the monthly payment. Lease payments must be made at the beginning of the period.

a. How much does Chris spend in total for monthly payments over the term of the financing for each option? Why is there a difference between the two amounts? Explain.

b. What is the total amount spent by Chris in each option if he would like to own the vehicle at the end of 36 months? What is the difference in final price? Explain.

c. Which option would you consider to be best for Chris and in which circumstances? Explain.

$\boxed{\text{SE}}$ **d.** How much would Chris have to save each month at 6%/a, compounded monthly, to pay for the \$10 051.64 buyout at the end of 36 months? Would this decrease his final cost for the lease or not? Explain.

Solution

a. **Option 1**
$660.71 \times 36 = 23\ 785.56$

Option 2
$407.87 \times 36 = 14\ 683.32$

The difference is that in option 1, Chris owns the car, and in option 2, Chris doesn't own the car.

b. The total amount spent by Chris in option 1 would be $25 785.56, which includes all payments plus the down payment of $2 000. The total cost for option 2 would be $26 734.96, which includes all payments, the down payment of $2 000, and the final buyout of $10 051.64. The difference in final price to purchase the car would be $949.40. It is assumed that Chris has the $10 051.64 for the final buyout and does not have to finance this portion as well.

c. If Chris can afford the monthly payment of $660.71 for 36 months, he will save $949.40. If Chris cannot afford the higher payment, then the lease value is better, but he must remember to save the money for the buyout over the next 36 months.

SE d. To save the buyout amount over 36 months at 6%/a, compounded monthly, Chris would have to save $255.53 per month or $9 199.08; this would amount to $10 051.64, including interest. This would cut his final cost to $25 882.40, which includes $2 000 down payment, $14 683.32 in lease payments, and $9 199.08 in savings for 36 months.

7. A person borrows \$15 000 to purchase a new car. The loan is for 36 months at 8%/a, compounded monthly, with payments of \$470.05/mo. Construct a spreadsheet that will allow the person to check the amount owing at any time during the three years.

Solution

	A	B	C	D	E	F
1						
2			Loan Information			
3						
4						
5	Period of Loan (months)	36				
6	Amount Borrowed	15 000				
7	Monthly Payment	470.05				
8	Interest Rate	0.08				
9						
10				Balance		New
11		Month	Interest	with Interest	Payment	Balance
12		1	=B6*(\$B\$8/12)	=B6+C12	=\$B\$7	=D12-E12
13		=B12+1	=F12*(\$B\$8/12)	=F12+C13	=\$B\$7	=D13-E13
14						
15						
16						
17						
18						
19						
20						
21						
22						
23						
24						
25						
26		***Use the "fill down" command to enter the formulas for the remainder of the spreadsheet.***				
27						
28						
29						
30						
31						
32						
33						
34						
35						
36						
37						
38						
39						
40						
41						
42						
43						
44						
45						
46						
47		=B46+1	=F46*(\$B\$8/12)	=F46+C47	=\$B\$7	=D47-E47
48						
49	**Totals**		=SUM(C12:C47)		=SUM(E12:E47)	
50						

The previous spreadsheet will result in the following values:

	A	B	C	D	E	F
1						
2			Loan Information			
3						
4						
5	Period of Loan (months)	36				
6	Amount Borrowed	$15 000.00				
7	Monthly Payment	$470.05				
8	Interest Rate	8.00%				
9						
10				Balance		New
11		Month	Interest	with Interest	Payment	Balance
12		1	$100.00	$15 100.00	$470.05	$14 629.95
13		2	$97.53	$14 727.48	$470.05	$14 257.43
14		3	$95.05	$14 352.48	$470.05	$13 882.43
15		4	$92.55	$13 974.98	$470.05	$13 504.93
16		5	$90.03	$13 594.96	$470.05	$13 124.91
17		6	$87.50	$13 212.41	$470.05	$12 742.36
18		7	$84.95	$12 827.31	$470.05	$12 357.26
19		8	$82.38	$12 439.65	$470.05	$11 969.60
20		9	$79.80	$12 049.39	$470.05	$11 579.34
21		10	$77.20	$11 656.54	$470.05	$11 186.49
22		11	$74.58	$11 261.06	$470.05	$10 791.01
23		12	$71.94	$10 862.95	$470.05	$10 392.90
24		13	$69.29	$10 462.19	$470.05	$9 992.14
25		14	$66.61	$10 058.76	$470.05	$9 588.71
26		15	$63.92	$9 652.63	$470.05	$9 182.58
27		16	$61.22	$9 243.80	$470.05	$8 773.75
28		17	$58.49	$8 832.24	$470.05	$8 362.19
29		18	$55.75	$8 417.94	$470.05	$7 947.89
30		19	$52.99	$8 000.87	$470.05	$7 530.82
31		20	$50.21	$7 581.03	$470.05	$7 110.98
32		21	$47.41	$7 158.38	$470.05	$6 688.33
33		22	$44.59	$6 732.92	$470.05	$6 262.87
34		23	$41.75	$6 304.63	$470.05	$5 834.58
35		24	$38.90	$5 873.47	$470.05	$5 403.42
36		25	$36.02	$5 439.45	$470.05	$4 969.40
37		26	$33.13	$5 002.53	$470.05	$4 532.48
38		27	$30.22	$4 562.69	$470.05	$4 092.64
39		28	$27.28	$4 119.93	$470.05	$3 649.88
40		29	$24.33	$3 674.21	$470.05	$3 204.16
41		30	$21.36	$3 225.52	$470.05	$2 755.47
42		31	$18.37	$2 773.84	$470.05	$2 303.79
43		32	$15.36	$2 319.15	$470.05	$1 849.10
44		33	$12.33	$1 861.43	$470.05	$1 391.38
45		34	$9.28	$1 400.65	$470.05	$930.60
46		35	$6.20	$936.81	$470.05	$466.76
47		36	$3.11	$469.87	$470.05	-$0.18
48						
49	Totals		$1 921.62		$16 921.80	
50						

(From Applied Mathematics 30 January 2001 Diploma Examination, Multiple-Choice Question 16)

Use the following information to answer the next question.

The following spreadsheet shows the beginning of an amortization table for 5 equal monthly payments to be made on a $900 loan with an interest rate of 1% per month.

	A	B	C	D	E
1	**Payment Number**	**Payment**	**Interest Payment**	**Payment to Principal**	**Balance Remaining**
2	1	$155.29	$9.00	$146.29	$753.71
3	2	$155.29	$7.54	$147.75	$605.96
4	3	$155.29			
5	4				
6	5				

8. Which of the following formulas can be used to calculate the value of cell D4?

A. =B4 + C4

B. =E3 – B4

C. =B4 – 0.1*E3

D. =B4 – 0.01*E3

Solution

Payment to principal is determined by subtracting the interest paid (1%) from the monthly payment. Therefore, a formula that will work is =B4 – 0.01 *E3.

(From Applied Mathematics 30 January 2001 Diploma Examination, Multiple-Choice Question 17)

Use the following information to answer the next question.

A person has invested $20 000 with an investment firm. In the first year, her portfolio gives the following returns:

Type	**Percent Invested**	**First-Year Profit**
Guaranteed certificates	20%	4%
Blue-chip stocks	50%	9.25%
High-risk stocks	30%	–7.5%

SE **9.** For this portfolio, the rate of return, to the nearest tenth of a percentage, is

A. 20.8%
B. 5.8%
C. 3.2%
D. 1.9%

Solution

$$\begin{aligned}\text{Amount of return} &= (20\,000\times0.20\times0.04)+(20\,000\times0.50\times0.0925)\\ &\quad(20\,000\times0.30\times0.075)\\ &= \$635\end{aligned}$$

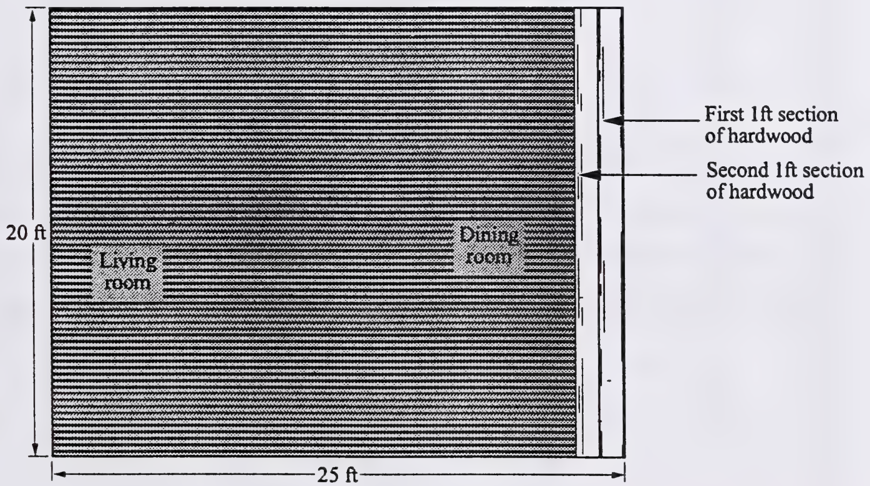
$$\begin{aligned}\text{Rate of return} &= \frac{\$635}{20\,000}\\ &= 0.03175\\ &= 3.5\%\end{aligned}$$

(From Applied Mathematics 30 January 2001 Diploma Examination, Written-Response Question 1)

Use the following information to answer the next question.

A family is planning to cover their 20 ft × 25 ft living room/dining room with carpet and hardwood. Their budget for this project is $5 000. The cost of the carpet is $7.25/ft^2, and the cost of the hardwood is $12.50/ft^2.

The family would like to cover the entire area with hardwood, but they know that it would cost more than the budgeted amount. In order to determine the maximum area of hardwood that they can afford, they calculated the total cost of the flooring each time a 1 ft wide section of hardwood was added to the room, as shown below.



The family used the following spreadsheet to calculate costs and to determine the maximum area of hardwood that they could afford.

	A	B	C	D	E
1	**Area of floor with hardwood (ft^2)**	**Area of floor with carpet (ft^2)**	**Cost of hardwood @ \$12.50/ft^2**	**Cost of carpet @ \$7.25/ft^2**	**Total cost of flooring**
2	500	0	\$6 250	\$0	\$6 250
3	480	20	\$6 000	\$145	\$6 145
4	460	40	\$5 750	\$290	\$6 040
5	440	60	\$5 500	\$435	\$5 935
6	420	80	\$5 250	\$580	\$5 830
7	400	100	\$5 000	\$725	\$5 725
8	380	120	\$4 750	\$870	\$5 620
9	360	140	\$4 500	\$1 015	\$5 515
10	340	160	\$4 250	\$1 160	\$5 410
11	320	180	\$4 000	\$1 305	\$5 305
12	300	200	\$3 750	\$1 450	\$5 200
13	280	220	\$3 500	\$1 595	\$5 095
14	260	240	\$3 250	\$1 740	\$4 990
15	240	260	\$3 000	\$1 885	\$4 885
16	220	280	\$2 750	\$2 030	\$4 780
17	200	300	\$2 500	\$2 175	\$4 675
18	180	320	\$2 250	\$2 320	\$4 570
19	160	340	\$2 000	\$2 465	\$4 465
20	140	360	\$1 750	\$2 610	\$4 360
21	120	380	\$1 500	\$2 755	\$4 255
22	100	400	\$1 250	\$2 900	\$4 150
23	80	420	\$1 000	\$3 045	\$4 045
24	60	440	\$750	\$3 190	\$3 940
25	40	460	\$500	\$3 335	\$3 835
26	20	480	\$250	\$3 480	\$3 730
27	0	500	0	\$3 625	\$3 625

Written Response—10%

1. **a.** Explain the relationship between the values in columns A and B.

A POSSIBLE SOLUTION to part a

The sum of each row of columns A and B yields 500 ft^2 (the total floor of the living room and dining room).

b. Show, by writing a statement or a formula, how the value in cell E9 ($5 515) was calculated. Make reference to other cells in row 9.

A POSSIBLE SOLUTION to part b
Cell C9 is calculated by multiplying A9, the total area of hardwood, by $12.50. Cell D9 is calculated by multiplying B9, the total area of carpet, by $7.25. Cell E9 is then calculated by adding cells C9 and D9 to find the total cost of flooring for the given areas of carpet and hardwood. **or** E9 = A9 * 12.50 + B9 * 7.25 **or** C9 = A9 * 12.50 D9 = B9 * 7.25 C9 + D9 = E9 **or** C9 + D9 = E9

c. If the family is to remain within their budget, what is the maximum area of hardwood that they can place into this living room/dining room area?

A POSSIBLE SOLUTION to part c
To remain within budget, the maximum area of hardwood is a $13 \text{ ft} \times 20 \text{ ft}$ piece or 260 ft^2.

d. • What is the total cost for this plan?

POSSIBLE SOLUTIONS to part d, bullet one
Option 1: The family could have 250 ft^2 each of carpet and hardwood for a total cost of \$4 937.50. **Option 2:** reference row 14 260 ft^2 of hardwood and 240 ft^2 of carpet for a total cost of \$4 990. **Option 3:** reference row 15 240 ft^2 of hardwood and 260 ft^2 of carpet for a total cost of \$4 885.

• Will the family remain within their budget? Explain.

A POSSIBLE SOLUTION to part d., bullet two
The total cost is less than \$5 000, so the family will remain within their budget.

Cyclic, Recursive, and Fractal Patterns

General Outcomes

Use patterns to describe the world and to solve problems.

Generate and analyze cyclic, recursive, and fractal patterns.

General Note:

- It is not the intent of this unit to study sinusoidal curves and characteristics of fractal patterns, but to use these, and other functions, as regression models to show patterns from which inferences can be drawn.
- These outcomes require substitution of calculator generated parameters into the regression equations and an understanding of the significance of these values.
- A fractal is a geometric figure exhibiting self-similarity and is produced by repeated iterations. Each successive fractal in a fractal pattern is more complex than the previous one.

Specific Outcomes

Specific Outcome 4.1

Collect sinusoidal data, graph the data using technology, and represent the data with a best fit equation of the form $y = a \sin(bx + c) + d$. [C, CN, PS, T, V]

4.1 Notes:

When using technology to generate a sinusoidal regression model, note the following points.

- A minimum of five points over most of one period is normally sufficient to produce an adequate regression model. Creating a list of eight or more points over two periods produces a better one. It is recommended that when entering values for sinusoidal regression into a graphing calculator, students should begin with the lowest x value.
- Graphing calculators will model sinusoidal regression in the form $y = a \sin(bx + c) + d$, where x is measured in radians. This means that the period of the function is $\frac{2\pi}{b}$ units and the horizontal transformation is to the left or right. To graph a sinusoidal regression model, students' calculators must be in radian mode. This means that teachers will have to explain to students that there are two ways to express angle measures: in radians and in degrees. It is not the intent to teach conversions from one measure to another, but simply to make students aware of the two methods. A visual representation showing the sector angle where the arc length is equal to the radius (depicting one radian measure) is sufficient. Students should be aware of the radian measure equivalents of 0°, 90°, 180°, 270°, and 360°.
- Sinusoidal regression models on a graphing calculator will occasionally have very small values for one or more of the parameters. Students will require practice recognizing these values and should write values less than or equal to 10^{-4} as zero.
- Recognize that a value of c other than zero is an indication that the initial point of the scenario is to the left or right of the *y-axis*.

- Determining the exact value of the horizontal transformation $\left(\frac{c}{b}\right)$ is beyond the scope of Applied Mathematics 30.
- The notation for a sinusoidal regression equation in Applied Mathematics 30 is $y = a \sin (bx + c) + d$, and in Pure Mathematics 30 it is $y = a \sin b(x + c)+d$.

Specific Outcome 4.2

Use best-fit sinusoidal equations and their associated graphs to make predictions (interpolation, extrapolation). [C, CN, PS, T]

4.2 Notes:

- Data showing trends in temperature changes can be found in most atlases.
- Students should be able to make predictions from equations and graphs.

Specific Outcome 4.3

Describe periodic events, including sinusoidal curves, using correct terminology. [C, V]

4.3 Note:

- Graphs should be provided in all questions relating to this outcome.

Specific Outcome 4.4

Use technology to generate and graph sequences that model real-life phenomena. [PS, T, V]

4.4 Note:

- Students should be given an opportunity to determine an equation, without the use of technology, that models a sequence.

Specific Outcome 4.5

Use technology to construct a fractal pattern by repeatedly applying a procedure to a geometric figure. [CN, R, V, T]

4.5 Notes:

- The purpose of this technology is to test conjectures without intensive labour. Students should, however, be able to generate the first few fractal patterns from simple figures such as regular polygons. Grid paper is essential to help students create accurate fractal shapes.
- Geometer's Sketchpad or a compass and straightedge may be appropriate for classwork but assessment should be with paper and pencil only. Formulas can be generated by regression models.
- Students should be familiar with regression models from Applied Mathematics 10 and 20 and should be able to determine which regression is the "best fit" for a particular set of data.
- Define iterations as repeated recursive calculations or procedures.

Specific Outcome 4.6

Use the concept of self-similarity to compare and/or predict the perimeters, areas, and volumes of fractal patterns. [CN, R, T, V]

4.6 Notes:

- Be aware that fractal patterns can become very difficult very quickly. Look for examples that can be modelled by exponential regression. This can usually be accomplished by focusing on the number of new shapes at each iteration rather than on the total number of shapes.
- Two important properties of fractal patterns are
  – self-similarity (characteristics of the shape are maintained under any magnification)
  – increasing complexity of the shape at each iteration
- For area and volume fractal patterns, the pattern does not establish itself between the original and the first iteration.
- The volumes resulting from successive iterations of three-dimensional fractal patterns do not form fractals. This fact may lead to interesting discussions if limited to fairly simple shapes with reasonable numbers.
- Exponential regression cannot be performed on fractal patterns involving increasing areas and volumes.

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• use technology to graph data that represents a sinusoidal curve	
• state appropriate graphing calculator window settings as an ordered triple in the form x: $[x_{\text{min}}, x_{\text{max}}, x_{\text{scale}}]$ and y: $[y_{\text{min}}, y_{\text{max}}, y_{\text{scale}}]$	
• use the regression model $y = \text{a} \sin(bx + c) + d$ to determine a best-fit equation	• provide explanations for trends in data
• use the TRACE or CALC function on a graphing calculator to make predictions	
• describe a periodic event given a diagram or graph	• describe the transformational effect of additional information on a periodic event
• make predictions from the derived sinusoidal regression equation	• sketch transformations of a given graph
• describe a periodic event using up to three terms (amplitude, period, maximum and minimum values, vertical and horizontal shifts)	• describe a periodic event using three or more terms (amplitude, period, maximum and minimum values, vertical and horizontal shifts, axis placements, and scaling)
• relate values of a, b, and d of $y = a \sin(bx + c) + d$ to given window settings	• choose appropriate window settings for a graphing calculator, based on given values for amplitude, period, and vertical shift
• determine the values of a, b, and d in the equation $y = a\,(bx + c) + d$ or in a given graph.	
• recognize that when $c \neq 0$ a horizontal transformation occurs	
• enter data into a list or scatter plot, apply a best-fit model (either linear, quadratic, or exponential), and use the regression equation to make predictions	
• distinguish between patterns that are possibly fractals and those that are clearly not fractals	
• generate the next two iterations of a fractal pattern, given the original shape and a written and visual description of the pattern	
• describe, in writing or by listing a sequence of numbers, the patterns related to length of sides and to number of sides or number of vertices of fractal shapes	
• describe the numerical pattern for the perimeter of each shape in a two-dimensional fractal pattern	
• determine an algebraic expression describing the perimeter of the shapes in a fractal pattern	• give a rule for the change in area for the first few iterations of a fractal pattern
• generalize a fractal pattern using an algebraic expression that represents the patterns in length of sides, number of sides, or number of vertices of fractal shapes	• find the volume of the shapes for the first few iterations of a simple fractal pattern and make conjectures

- participate in and contribute toward the problem-solving process for problems that require the analysis of patterns and fractals studied in Applied Mathematics 30
- complete the solution to problems that require the analysis of patterns and fractals studied in Applied Mathematics 30

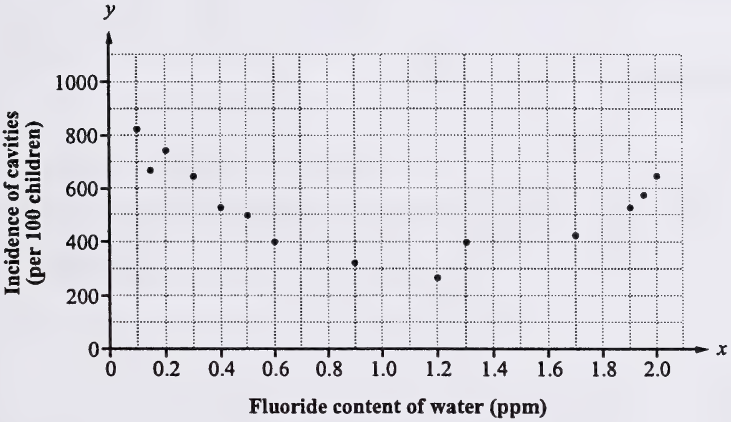
Examples

Students who achieve the *Acceptable Standard* should be able to answer all the following questions, except for any part of a question labelled $\boxed{\text{SE}}$. Parts labelled $\boxed{\text{SE}}$ are appropriate examples for students who achieve the *Standard of Excellence.*

(From Applied Mathematics 30 January 2001 Diploma Examination, Multiple-Choice Question 10)

Use the following information to answer the next question.

The scatter plot below compares the incidence of cavities per 100 children and the fluoride content of water in parts per million (ppm) in a particular area.



1. Which of the following regression equations is **most appropriate** for the given data?

A. Linear
B. Quadratic
C. Sinusoidal
D. Exponential

2. The following data were entered into two lists and a sinusoidal regression was performed on the data.

(0, 0)	The value of each parameter was calculated as follows:
(5, 28.27)	
(17, 18.21)	
(24, –23.45)	$a = 40.01018508$
(32, –38.07)	$b = 0.1570800685$
(40, 0)	$c = -8.802945\text{E} - 4$
	$d = 0.0200849825$

State the regression equation in the form $y = a \sin(bx + c) + d$, and round all values to the nearest thousandth.

Solution

$y = 40.010 \sin(0.157x + 0.000) + 0.020$

3. The following table shows the average daily temperature by month in Winnipeg, Manitoba, over a period of one year.

Average Daily Temperature in Winnipeg, Manitoba	
Month	**Average Temperature**
January	–18
February	–18
March	–11
April	0
May	11
June	18
July	20
August	19
September	14
October	7
November	–8
December	–17

a. Use a graphing calculator to plot the data.

b. Determine an approximate sinusoidal function for this data. Express the values to the nearest hundredth. Graph this function in the viewing window provided and state the window settings.

x: [, ,]

y: [, ,]

function: ____________________________

c. Use the regression model to determine the average temperature on August 15.

d. What is the period of this function? How does it compare with your knowledge of annual temperature changes?

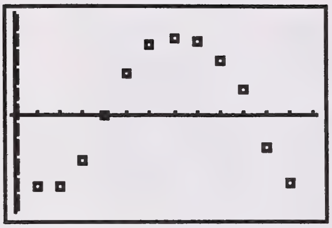
e. What do the parameters a and d in your regression equation represent in terms of the annual mean temperature and range of temperatures in Winnipeg, Manitoba?

f. The growing season is the part of the year in which the average temperature remains above 5°C. What is the growing season in Winnipeg? Justify your answer using data from the graph or equation above.

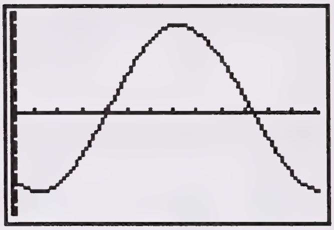
g. If the average daily temperature rose by 2°C as a result of global warming, what would be the effect on the sinusoidal function? What would be the new growing season? Explain.

Solution

a. A possible solution



b.



$x: \begin{bmatrix} 0, & 13.1, & 1 \end{bmatrix}$

$y: \begin{bmatrix} -25, & 25, & 1 \end{bmatrix}$

function: $\underline{y = 20.61 \sin(0.52x - 2.16) + 1.21}$

c. There are 31 days in August, so for August 15, $x = 8.48$ (approximately).
When $x = 8.48$, $y = 17.5$. Therefore, the average temperature on August 15 is 17.5°C.

d.
$$\text{period} = \frac{2\pi}{b}$$
$$b = 0.51799...$$
$$\frac{2\pi}{0.51799...} = 12.1298...$$
The period of this function is 12.1. This value seems reasonable since the temperatures should follow this pattern every year.

e. The value of a is 20.60... and the value of d is 1.20.... This indicates that the annual mean temperature in Winnipeg is about 1.21°C. The range of the temperature should be from as high as $1.21 + 20.61 = 21.82$°C to as low as $1.21 - 20.61 = -19.40$°C.

f. Graph the line $y = 5$°C on the same axis as the regression equation. The sinusoidal curve lies above $y = 5$ between $x = 4.5$ and $x = 9.9$. These points translate to April 15 and September 27. Therefore, the growing season is from April 15 to September 27.

g. The value of d in the sinusoidal function would increase by 2, so the new equation would be
$$y = 20.61 \sin(0.52x - 2.16) + \mathbf{3.21}$$

The points of intersection of this curve and $y = 5$ are $x = 4.3$ and $x = 10.1$. The new growing season would be from April 9 to October 3.

4. A mild infection spreads through a school with a population of 1 000 students. On any given day, 10% of the uninfected population becomes infected, and 20% of the infected population is "cured." Once cured, there is zero chance of further infection. On the first day of school, 100 students are infected, 900 students are uninfected, and 0 students are cured. From this information, the following data has been generated.

Day	Uninfected	Infected	Cured	Sum
1	900	100	0	1 000
2	810	170	20	1 000
3	729	217	54	1 000
4	656	247	97	1 000
5	590	263	147	1 000
6	531	269	199	1 000
7	478	269	253	1 000
8	430	263	307	1 000
9	387	253	359	1 000
10	349	241	410	1 000
11	314	228	458	1 000
12	282	214	504	1 000
13	254	199	547	1 000
14	229	185	586	1 000
15	206	171	623	1 000

a. Graph the infected population as a function of time over 15 days, and state the window settings you used.

b. Perform a quadratic regression and state your regression equation. Express the values of the parameters to the nearest hundredth.

$\boxed{\text{SE}}$ **c.** The epidemic is considered to be under control when the percentage of the population that is infected drops below 10%. Use your quadratic regression model to predict which day the epidemic is considered under control. Comment on the reasonableness of this answer.

Solution

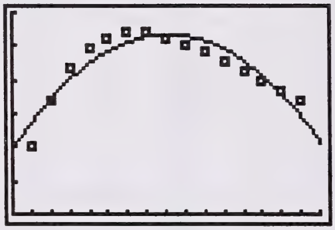
a. Data can be plotted on a window of

$x: \begin{bmatrix} 0, & 16, & 1 \end{bmatrix}$

$y: \begin{bmatrix} 0, & 300, & 50 \end{bmatrix}$

b. Regression equation: $y = -2.56x^2 + 41.70x + 97.65$

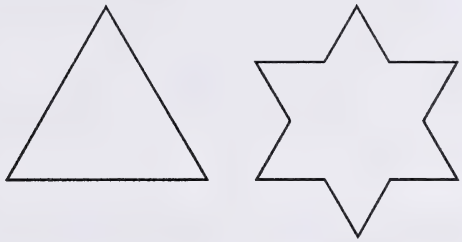
SE c. According to the quadratic regression, the epidemic will be under control on day 17. This is not a reasonable answer because the regression model is a poor fit to the data, as can be seen below.



Note: The data could also be generated using the following spreadsheet program.

	A	B	C	D	E
1	Day	Uninfected	Infected	Cured	Check
2	1	900	100	0	=B2+C2+D2
3	=1+A2	=(B2*0.9)	=(C2*0.8)+(B2*0.1)	=C2*0.2	=B3+C3+D3
4	=1+A3	=(B3*0.9)	=(C3*0.8)+(B3*0.1)	=C3*0.2+D3	=B4+C4+D4
5	=1+A4	=(B4*0.9)	=((C4*0.8)+(B4*0.1))	=(C4*0.2+D4)	=B5+C5+D5
6	=1+A5	=(B5*0.9)	=((C5*0.8)+(B5*0.1))	=(C5*0.2+D5)	=B6+C6+D6
7	=1+A6	=(B6*0.9)	=((C6*0.8)+(B6*0.1))	=(C6*0.2+D6)	=B7+C7+D7
8	=1+A7	=(B7*0.9)	=((C7*0.8)+(B7*0.1))	=(C7*0.2+D7)	=B8+C8+D8
9	=1+A8	=(B8*0.9)	=((C8*0.8)+(B8*0.1))	=(C8*0.2+D8)	=B9+C9+D9
10	=1+A9	=(B9*0.9)	=((C9*0.8)+(B9*0.1))	=(C9*0.2+D9)	=B10+C10+D10
11	=1+A10	=(B10*0.9)	=((C10*0.8)+(B10*0.1))	=(C10*0.2+D10)	=B11+C11+D11
12	=1+A11	=(B11*0.9)	=((C11*0.8)+(B11*0.1))	=(C11*0.2+D11)	=B12+C12+D12
13	=1+A12	=(B12*0.9)	=((C12*0.8)+(B12*0.1))	=(C12*0.2+D12)	=B13+C13+D13
14	=1+A13	=(B13*0.9)	=((C13*0.8)+(B13*0.1))	=(C13*0.2+D13)	=B14+C14+D14
15	=1+A14	=(B14*0.9)	=((C14*0.8)+(B14*0.1))	=(C14*0.2+D14)	=B15+C15+D15
16	=1+A15	=(B15*0.9)	=((C15*0.8)+(B15*0.1))	=(C15*0.2+D15)	=B16+C16+D16

5. The following is an example of a Koch snowflake. This fractal pattern is constructed by starting with an equilateral triangle, trisecting each side, and constructing an equilateral triangle in each middle third. The pattern is continued to produce the fractal pattern. The original iteration and iteration 1 are shown below.



Iteration 0 (Original) Iteration 1

a. Continue the pattern to construct iteration 2.

b. The length of each side in iteration 0 is 9 cm. State the relationship between the length of the sides in each iteration.

c. Predict the length of each side of the snowflake in iteration 3.

d. Write an equation that relates the length of each side to the number of iterations.

e. State the relationship between the number of sides of the snowflake in each iteration.

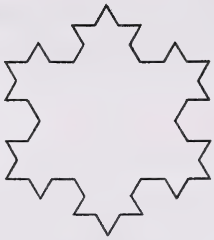
f. Predict the number of sides in iteration 3.

g. Write an equation that relates the number of sides to the number of iterations.

h. Calculate the area of the original iteration and the snowflake created in iteration 1.

Solution

a.



b. Iteration 0: 9 cm
Iteration 1: 3 cm
Iteration 2: 1 cm
Each iteration produces shapes with sides that are one-third the length of those in the previous iteration.

c. The third iteration will have sides $\frac{1}{3}$ cm long.

d. $y = 9\left(\frac{1}{3}\right)^x$, where y = length of sides
x = number of iteration

e. Iteration 0: 3 sides
Iteration 1: 12 sides
Iteration 2: 48 sides
The number of sides is multiplied by 4 in each iteration.

f. $48 \times 4 = 192$ sides in iteration 3

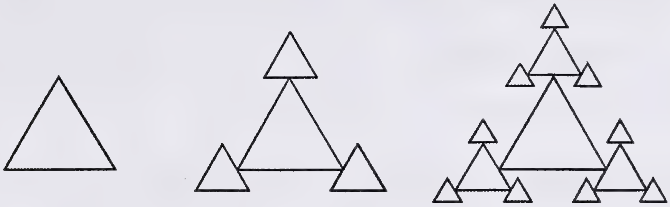
g. $y = 3(4)^x$
y = number of sides
x = iteration number

h. Iteration 0: $A = \frac{1}{2}(9)\left(\sqrt{9^2 - 4.5^2}\right) = 35.07 \text{ cm}^2$

Iteration 1: $A = 3\left[\frac{1}{2}(3)\left(\sqrt{3^2 - 1.5^2}\right)\right] + \frac{1}{2}(9)\left(\sqrt{9^2 - 4.5^2}\right) = 46.77 \text{ cm}^2$

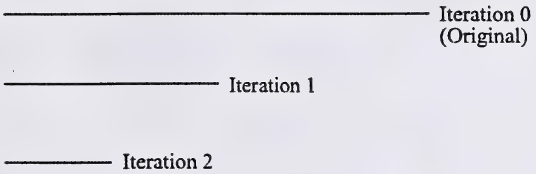
6. Determine which of the following patterns are examples of fractals. Explain your answer.

a.

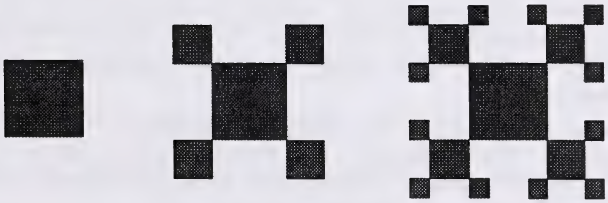


Iteration 0 (Original) | Iteration 1 | Iteration 2

b.



c.



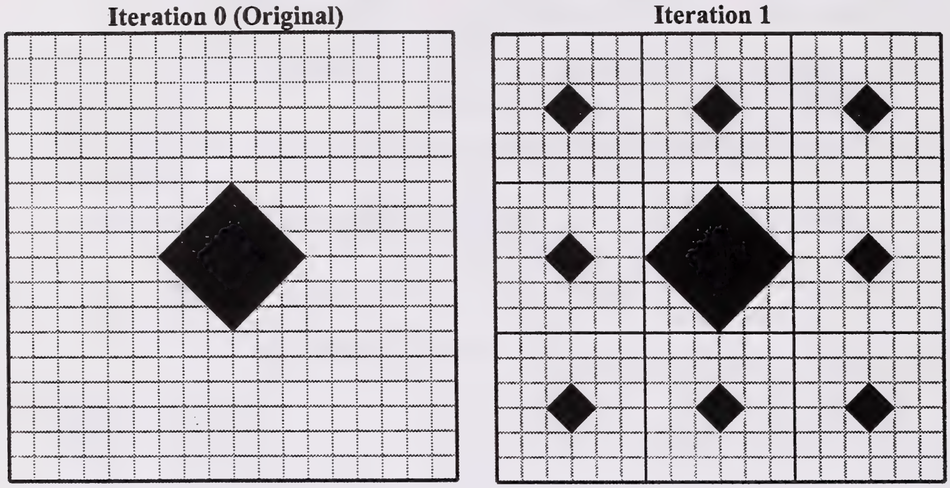
Iteration 0 (Original) | Iteration 1 | Iteration 2

Solution

a. A fractal pattern. Each iteration is more complex and exhibits self-similarity.
b. Not a fractal pattern. Each iteration is not more complex than the previous one.
c. Not a fractal pattern. The iterations do not exhibit self-similarity.

7. The general rule for the fractal pattern shown below is to start with a square and remove a rhombus.

   a. Describe the rule that was used to determine the size of the rhombus that was removed in iteration 0.
   b. Compare the first two iterations and describe the rule that was used to construct the second iteration from the first.
   c. Construct iteration 2 by applying the rule that you stated.



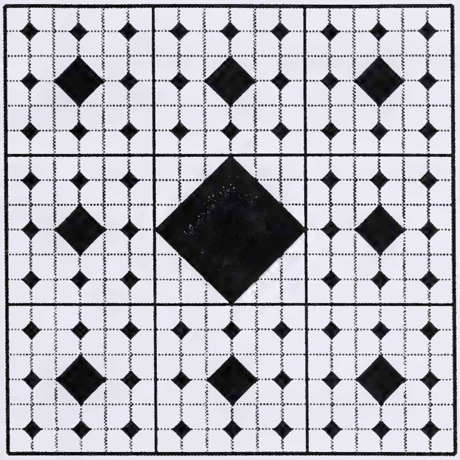
   d. Examine the three iterations, and record the length of the sides of each of the rhombi removed. Compare the length of the side of the rhombi created in iteration 2 with those in iteration 1 and iteration 0.
   e. Predict the length of the sides of the rhombi removed in iteration 3.
   f. Refer back to the original. What is the **area** of the rhombus that was removed?
   g. Determine the area of each rhombus that was removed in iteration 1 and 2.
   h. Predict the area of each rhombus to be removed in iteration 3?

$\boxed{\text{SE}}$ i. State the rule used to determine the area of the rhombus to be removed in the n^{th} iteration.

Challenge: Find the perimeter of all the rhombi in iteration 2.
Find the area of the figure that remains once all the rhombi are removed in iteration 2.

Solution

a. Each length of the sides of the large square is divided into thirds. The midpoint of the middle thirds are found and joined to form a rhombus. This shape is removed.
b. In each of the remaining small squares, divide the sides into thirds, find the midpoints and join to form a rhombus, and remove the middle third.
c. Iteration 2



d. The sides of the rhombi in the three iterations are

$\sqrt{18}$, $\sqrt{2}$, and $\frac{\sqrt{2}}{3}$ units long. The length of the sides in iteration 2 is one-third the length of the sides in iteration 1 and one-ninth the length of the sides of the original.

e. The lengths of the sides of the rhombi removed in iteration 3 will be

$\frac{\sqrt{2}}{9}$ units.

f. The area of the rhombus removed from the original is 18 u^2.

g. The rhombus in iteration 1 has an area of $2\ u^2$.

The rhombus in iteration 2 has an area of $\frac{2}{9}\ u^2$.

In descending order, the areas are $18\ u^2$, $2\ u^2$, and $\frac{2}{9}\ u^2$.

h. In iteration 3, the area of each rhombus to be removed will be $\frac{2}{81}\ u^2$.

$\boxed{\text{SE}}$ **i.** The area of the rhombus removed in the n^{th} iteration is $\frac{1}{9}$ of the area of the previous rhombus. **Or** $y = A\left(\frac{1}{9}\right)^x$, where y = area removed, A is area of original, and x is iteration number.

Challenge: Perimeter of all the rhombi in iteration 2:

$$1\left(\sqrt{18} \times 4\right) + 8\left(\sqrt{2} \times 4\right) + 64\left(\frac{\sqrt{2}}{3} \times 4\right) = 182.90 \text{ units}$$

$$\begin{aligned}\text{Area of figure that remains} &= \text{original areas} - \text{removed area} \\ &= 324 - \left[1(18) + 8(2) + 64\left(\frac{2}{9}\right)\right] \\ &= 324 - 48\frac{2}{9} \\ &= 275\frac{7}{9}\text{u}^2\end{aligned}$$

(From Applied Mathematics 30 January 2001 Diploma Examination,Written-Response Question 3)

Use the following information to answer the next question.

The diameter of wire, in millimetres, is described by a gauge number. The following table relates selected gauge numbers and the corresponding wire diameter.

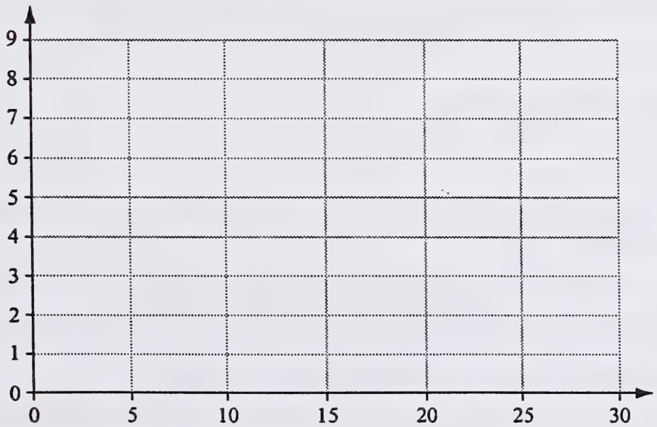
Gauge Number	Diameter of Wire (in mm)
0	8.25
5	4.62
10	2.59
15	1.45
20	0.81

Written Response—15 %

3. **a.** • Input the data above into two of your calculator lists, and graph the data with the window settings

x: [0, 30, 5]
y: [0, 9, 1]

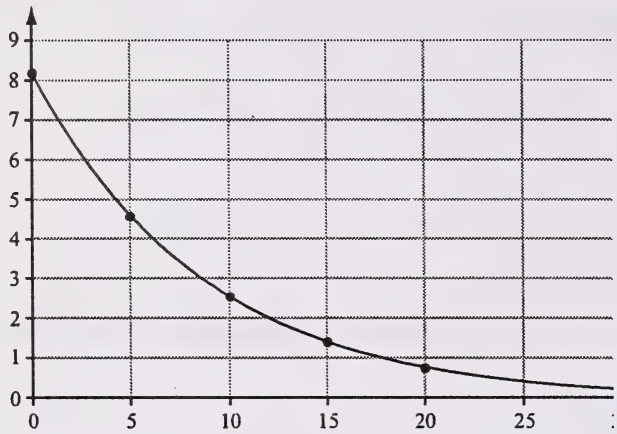
• Plot the information from your graphing calculator on the coordinate plane below.



b. • Perform an exponential regression on the data and sketch this regression model on the coordinate plane on the previous page.

• State the exponential regression equation in the form $y = ab^x$. Round the values of a and b to the nearest hundredth.

A POSSIBLE SOLUTION to parts a and b



$y = 8.25(0.89)^x$

c. What do the variables x and y represent in the context of this question?

A POSSIBLE SOLUTION to part d

x represents the wire gauge.
y represents the diameter of the wire (in mm).

d. Determine the diameter of wire for a 40-gauge wire, to the nearest hundredth of a millimetre.

A POSSIBLE SOLUTION to part c

The diameter of wire for a 40-gauge wire is 0.08 mm.

Use the following information to answer the next part of the question.

The forces of a drag on a car are measured by how much the speed drops during a 10-s interval of coasting without power. The results of some time trials are shown below.

Speed (km/h)	**Drag force (N)**
20	290
30	300
40	340
50	380
60	430

e. Use your graphing calculator to perform an exponential regression on this data. Compare the graph of this relationship with the graph relating wire gauge and diameter.

A POSSIBLE SOLUTION to part e

Both are exponential functions.
The first function falls to the right, whereas the second function rises to the right.
The y-intercept of the first function is 8.25, and y-intercept of the second function is 228.5.

Vectors

General Outcomes

Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Solve problems involving polygons and vectors, including both 3-D and 2-D applications.

General Notes:

- Students should be reminded to ensure that their calculator is in degree mode before doing problems in this unit.
- Before starting the unit, it is strongly recommended that teachers review with their students Sine Law, Cosine Law, and right-angle trigonometry from Applied Mathematics 10 and bridge the trigonometry problems to appropriate vector terminology and diagrams.
- The concept of the alternate and corresponding angles related to transversal and parallel lines is crucial to succeed in this unit and will need to be reviewed.
- Students will need to understand the difference between scalar quantities, such as distance and speed, which identify only magnitude (e.g., 50 m, 100 km/h) and vector quantities, such as displacement and velocity, which identify direction as well as magnitude (e.g., 100 km/h in a direction 10° south of east).
- Teachers may want to make a connection with Physics 20 and Science 10 concepts. The mathematical process of visualization should be addressed when working with vectors. For example, when doing vector addition, students should draw the vectors. This can be done using technology, prebuilt vector programs, and/or student modelling of vectors of displacement, velocity, etc.
- *Geometer's Sketchpad*, *Calculator-Based Rangers*, isometric dot paper, and engineering paper may all be helpful tools in teaching this unit.
- Application problems will be limited to force, displacement, and velocity.
- The vector unit is based on applications of right-angle and oblique triangle trigonometry.
- Most problems are applications of the primary trigonometric ratios and the Law of Cosines.
- In general, questions in which a diagram is provided will be deemed to be at the *Acceptable Standard*, and questions in which no diagram is given will be deemed to be at the *Standard of Excellence.*
- Angle measurements for direction of vectors will be dealt with in the context of the situation. Students should be prepared to work with both bearing notation (clockwise from north) and directional (heading) notation (W35°S means 35° south of west). We assume north to always be pointing to the top of the page.
- Unless otherwise stated, angle measures should be stated to the nearest degree and resultant magnitudes to the nearest tenth.

Specific Outcomes

Specific Outcome 5.1
Use appropriate terminology to describe
– vectors, i.e., magnitude, direction
– scalar quantities, i.e., magnitude [C, CN]

5.1 Note:
- Use an arrow for direction and length of relative magnitude.

Specific Outcome 5.2
Assign meaning to the multiplication of a vector by a scalar. [CN]

5.2 Note:
- Students should be encouraged to use visualization and scale models when working with vectors.

Specific Outcome 5.3
Determine the magnitude and direction of a resultant vector, using triangle or parallelogram methods. [CN, R, T, V]

5.3 Notes:
- Students should be comfortable using both parallelograms and triangles (head-to-tail) to solve problems involving vectors.
- When a force acts on a body, it should be represented by a vector whose initial point is the body and whose terminal point is in the direction that the force will move away from the body.
- Students will require a great deal of practice drawing appropriate vector diagrams and resultant vectors (both head-to-tail diagrams and parallelograms).
- The intent of the study of vectors in Applied Mathematics 30 is that students become aware that vectors represent real values, and therefore, resultant vectors should make sense physically.
- Students need to be made aware that the trigonometric solution they calculate for a problem may need to be **adapted** to the context of the vector problem. Refer to example 7 for clarification.

Specific Outcome 5.4

Model and solve problems in 2-D and simple 3-D, using vector diagrams and technology. [CN, PS, T, V]

5.4 Notes:

- Students at the *Acceptable Standard* should only be presented with illustrated 3-D problems that use multiple right triangles in two dimensions. Refer to example 7 for clarification.
- Teachers may wish to use rectangular prisms to help students model 3-D vectors.

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• identify properties of a parallelogram and use the properties to draw appropriate vector diagrams • construct an appropriate vector diagram from given information • describe vectors and scalar quantities using appropriate terminology • distinguish between a scalar quantity and a vector quantity • determine relative magnitude of vectors by comparing lengths • recognize that multiplying a vector by a positive scalar changes magnitude only • recognize that multiplying a vector by a negative scalar reverses direction • solve problems involving scalar multiplication	
• calculate magnitude and direction of a resultant vector, given a diagram	• calculate magnitude and direction of a resultant vector without being given a diagram
• solve 2-D problems in which a diagram is given	• solve 2-D problems without being given a parallelogram or head-to-tail diagram
• model 3-D problems	• model and solve 3-D problems
• solve simple 3-D problems that have been represented by two 2-D diagrams (one in a horizontal plane and one in a vertical plane)	
• participate in and contribute toward the problem-solving process for problems that require the analysis of vectors studied in Applied Mathematics 30	• complete the solution to problems that require the analysis of vectors studied in Applied Mathematics 30

Examples

Students who achieve the *Acceptable Standard* should be able to answer all the following questions, except for any part of a question labelled $\boxed{\textbf{SE}}$. Parts labelled $\boxed{\textbf{SE}}$ are appropriate examples for students who achieve the *Standard of Excellence*.

1. The vector $\vec{a}$ represents a velocity of 40 km/h east. Make a scale drawing of each of the following vectors.

a. $3\vec{a}$

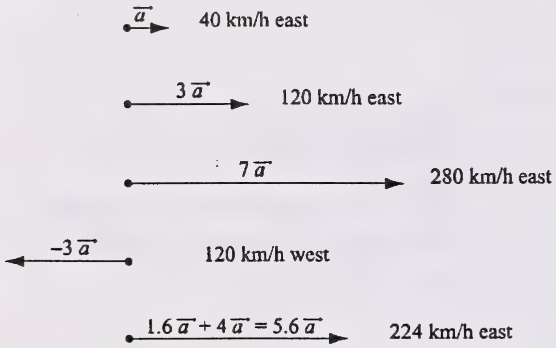
b. $7\vec{a}$

c. $-3\vec{a}$

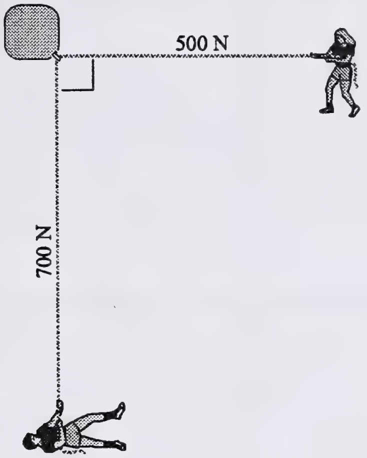
d. $1.6\vec{a} + 4\vec{a}$

What velocities do each of these vectors represent?

Solution



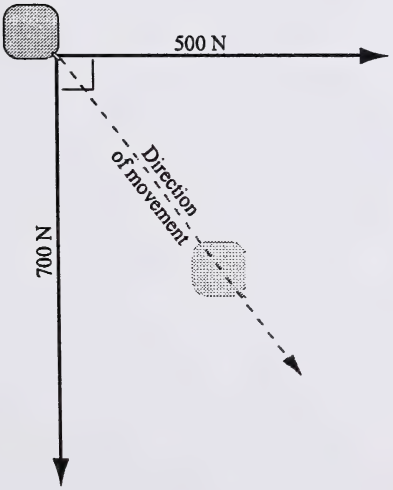
2. Two students are using ropes to pull on a heavy object, as shown in the diagram below.



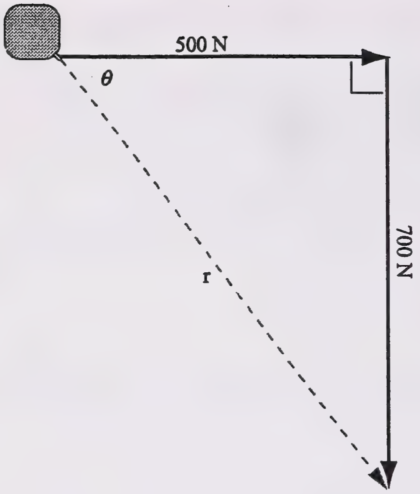
a. Sketch a diagram that will indicate the direction in which the object will move.
b. Draw a vector diagram that will show the movement and direction of the object, and solve for the angle, relative to the 500 N force, to the nearest degree.

Solution

a.



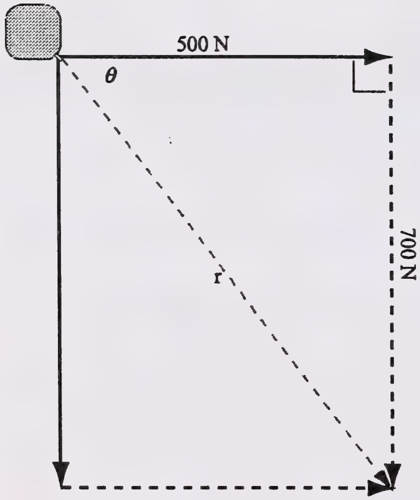
b.



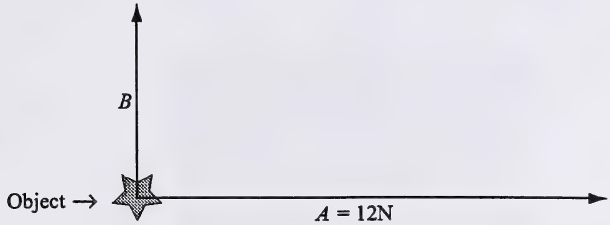
$$\tan\theta = \frac{700}{500}$$
$$\theta = 54.46°$$
$$\theta = 54°$$

The object will move 54° south of the 500 N force.

or

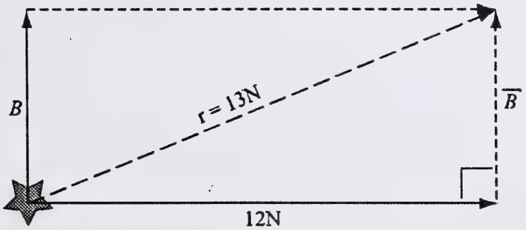


3. Person A is exerting 12 N of force as he pulls on an object. Person B is pulling perpendicular to person A, as modelled below.



Determine the amount of force that person B should exert in order to have the resultant force on the object equal to 13 N?

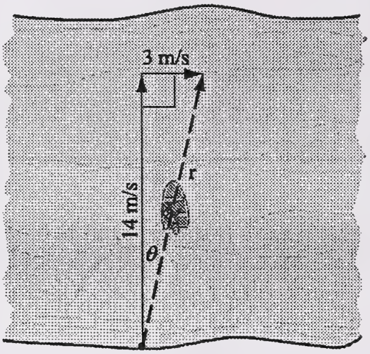
Solution



$$\begin{aligned} 13^2 &= 12^2 + B^2 \\ 169 &= 144 + B^2 \\ 25 &= B^2 \\ B &= 5 \end{aligned}$$

Person B should exert 5 N of force.

4. A boat is travelling across a river with a forward velocity of 14 m/s relative to the ground, and there is a current of 3 m/s down the river, as shown in the diagram below. What is the resultant speed of the boat?

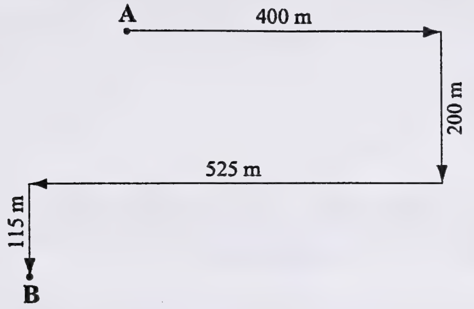


Solution

$$\sqrt{3^2 + 14^2} = r$$
$$14.3 = r$$

The boat is travelling 14.3 m/s.

5. A person walks along a path from point **A** to point **B**, as shown below.

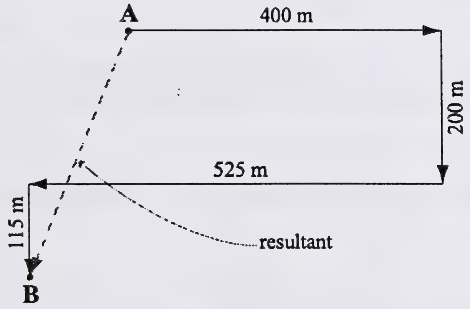


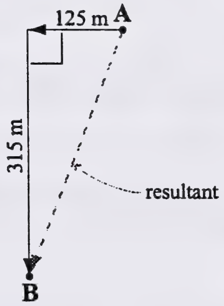
a. Draw an appropriate vector diagram for this path.

b. Determine the direction and straight-line distance that the person is from point **A** when she is at point **B**.

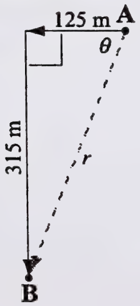
Solution

a.





b. Method One:



$$\tan\theta = \frac{315}{125}$$

$$\theta = 68°$$

$$315^2 + 125^2 = r^2$$

$$r = 338.9 \text{ m}$$

The person travels to a position 339 m W68°S of her initial position.

Method Two:
Using Rectangular Components:
$(400, 0) + (0, -200) + (-525, 0) + (0, -115) = (-125, -315)$
The resultant distance can then be calculated by using
$r^2 = (-125)^2 + (-315)^2$
$r^2 = 114\,850$
$r = 338.9$ m

The direction of movement in relation to the positive x-axis can be calculated by converting rectangular coordinates to polar coordinates. This results in an angle of $-111.6°$. An angle of $-112°$ is equal to W68°S.

Note: Some calculators on the list of approved calculators have the capability of performing this type of calculation. Students familiar with this method are free to use the technique; however, no questions on the diploma examination will specifically require this method of solution.

Use the following information to answer the next question.

Three forces are simultaneously acting on an object. The first force is 1 200 N upward, the second force is 700 N to the east, and the third force is 500 N to the west.

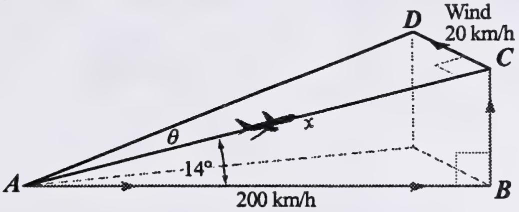
6. If it is assumed that the object will move as a result of these three forces, then what is the direction in which it will travel?

Solution

The object will move up, and since the force pulling toward the east is stronger than the force pulling to the west, it will move east.

Therefore, the object will move up and to the east.

7. A plane sets a course to fly with a ground speed of 200 km/h due east while climbing at an angle of 14°. Upon takeoff, it is affected by a 20 km/h wind blowing directly north, as shown in the diagram below.



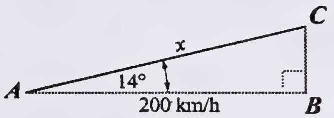
a. Model this problem using two 2-D diagrams.

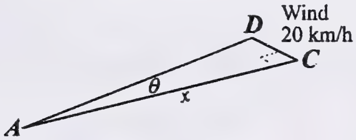
b. Calculate the speed (x) of the plane and calculate the direction (θ) that the plane is flying off its course.

c. What course correction should the pilot set to remain on track?

Solution

a.





b.

$$\cos 14° = \frac{200}{x}$$

$$x = \frac{200}{\cos 14°}$$

$$x = 206.1227\ldots \text{ km/h}$$

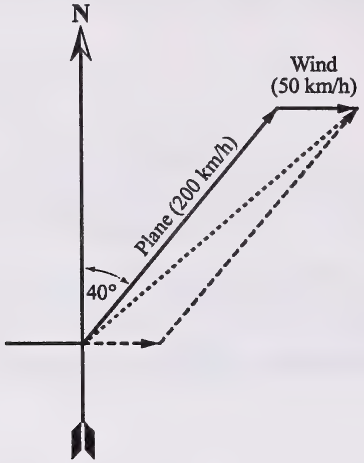
$$\tan\theta = \frac{20}{206.1227\ldots}$$

$$\theta = 5.5°$$

The plane is flying 206.2 km/h. It is flying off its course by E5.5°N (5.5° north of east).

c. To correct the path, the pilot should set a course correction of E5.5°S (5.5° south of east).

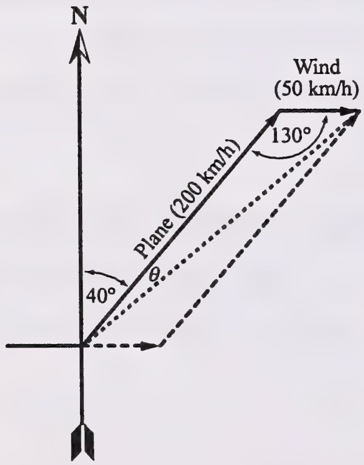
8. A small aircraft flying with an airspeed of 200 km/h on a bearing of 40° is being affected by a 50 km/h wind blowing east, modelled below.



a. Calculate the speed of the plane and the direction that the plane is flying off course.
b. Determine the velocity of the plane relative to the ground.

Solution

a.



Speed:

$$r = \sqrt{200^2 + 50^2 - 2(200)(50)\cos 130°}$$

$$r = 235.2780...$$

speed: 235.3 km/h

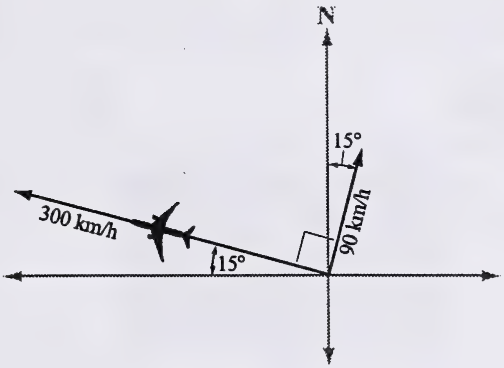
Direction:

$$\frac{\sin\theta}{50} = \frac{\sin 130°}{235.2780...}$$

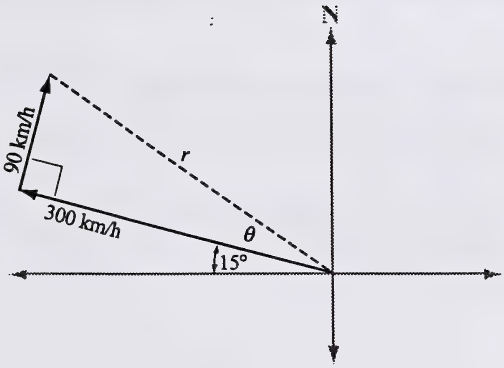
$$\theta = 9.4°$$

b. Relative to the ground, the velocity of the plane is 235.3 km/h on a bearing of 49°.

SE 9. An aircraft flying horizontally on a bearing of 285° is being pushed by a wind blowing N15°E. The indicated air speed of the aircraft is 300 km/h. The wind is constant at 90 km/h. After 1 hour of flight, what will be the aircraft's location relative to its initial position?



Solution



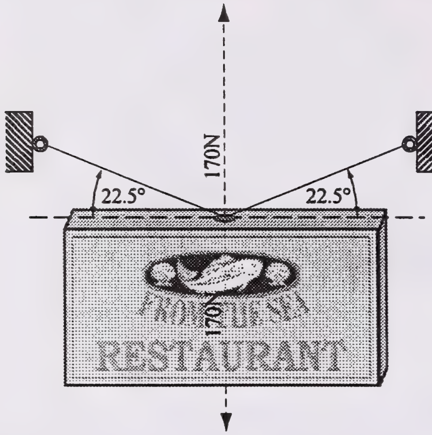
$$r^2 = 90^2 + 300^2$$
$$r^2 = 98\ 100$$
$$r = 313.21$$

$$\tan\theta = \frac{90}{300}$$

$$\theta = 16.7°$$
$$\therefore \angle = 285 + 16.7 = 301.7°$$

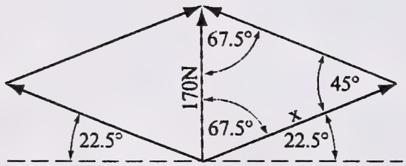
From its initial position, the plane will have flown 313.2 km on a bearing of 302°.

SE **10.** A sign supported by two wires of **equal length** exerts a downward force of 170 N. Both wires are attached to the sign at an angle of 22.5° to the horizontal, as shown in the diagram below.



Determine the amount of force, to the nearest newton, that is being exerted on each of the two wires.

Solution



$$\frac{\sin 67.5^\circ}{x} = \frac{\sin 45^\circ}{170}$$

$$\frac{170 \sin 67.5^\circ}{\sin 45^\circ} = x$$

$$222.12 = x$$

The force on each wire is 222 N.

Statistics and Probability

General Outcomes

Use normal and binomial probability distributions to solve problems involving uncertainty.

Use experimental or theoretical probability to represent and solve problems involving uncertainty.

General Notes:
- Students may need to review basic concepts and terms of probability.
- Teachers may want to do the probability section of this unit first.
- Probability should be expressed as a decimal or fractional value between 0 and 1.

Specific Outcomes

Specific Outcome 2.1
Find the population standard deviation of a data set or a probability distribution, using technology. [CN, E, T, V]

2.1 Notes:
- Review measures of central tendency and histograms. Teachers may also need to review the difference between a sample and a population.
- Sample standard deviation will not be used. Population standard deviation is represented by σ.
- A probability distribution can be displayed as a histogram that gives the probability for every outcome of an experiment. The probability distribution can then be converted into a data set.

Specific Outcome 2.2
Use z-scores and normal distribution to solve problems. [PS, R, T, V]

2.2 Notes:
- Some graphing calculators on the approved list do not have the capability to graph z-score probability curves; with these models of calculators, tables must be used.
- Students should have practice with paper and pencil calculations and sketches before being introduced to technology.
- Teachers and students need to be aware that the z-scores and areas under the curve from the textbook and data table may vary slightly from those values given by the calculator. When writing diploma examinations, students may use either technology **or** z-score tables.

Note: The z-score table that will be used on the Applied Mathematics 30 Diploma Examination follows a different format than those previously published by Alberta Learning.

Specific Outcome 2.3
Use the normal approximation to the binomial distribution to solve problems involving confidence intervals for large samples. [CN, E, PS]

2.3 Notes:
- A sample is considered to be large if $np > 5$ or if $n(1-p) > 5$. Continuity correction will not be assessed on the diploma examination.
- This outcome addresses only **discrete** data and is not intended for continuous variables.
- Calculation of the margin of error is **not** included in this outcome.

Specific Outcome 2.4
Construct a sample space for two or three events. [PS, R, V]

2.4 Notes:
- Some discrete sample spaces can also be displayed as the intersection points on a two-dimensional grid. See the solution of example 8 for an illustration.
- Once a sample space has been constructed, students should be able to answer questions based on it by using independent probability calculations or through investigation.
- It is not expected that students perform dependent probability calculations.

Specific Outcome 2.5
Classify events as independent or dependent. [C]

2.5 Notes:
- Most students should experience success with this outcome.
- The distinction between independent and dependent events should be approached through visualization of experimental design rather than through calculations.

Specific Outcome 2.6
Use expressions for P(A and B) to solve problems involving independent and dependent events. [CN, PS, R]

Specific Outcome 2.7
Solve problems using the probabilities of mutually exclusive and complementary events. [CN, PS, R]

2.7 Notes:
- The formulas for $P(A \text{ or } B)$ should be restricted to the mutually exclusive case only.
- Non-mutually exclusive events are beyond the scope of Applied Mathematics 30.

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• determine standard deviation and mean using hand-held technology	
• use technology to create a histogram, and compare it with a normal distribution	
• given various histograms, rank the standard deviation without doing any calculation	• interpret how changing the data can affect the standard deviation and/or mean
• calculate a z-score, using the formula	
• calculate the missing value when given the value of 3 of the 4 variables in the z-score formula	
• sketch a diagram of a normal curve and indicate appropriate shading for a given problem	
• given the values of n and p, calculate a z-score and apply it to the normal curve	• use z-scores to compare two sets of data and draw conclusions
• calculate the area under the standard normal curve to the left of a z-score	• calculate the area to the right of a z-score or between two z-scores
• describe the properties of a normal distribution	• make inferences given an area under the standard normal curve
• given a binomial distribution and using appropriate formulas, calculate mean and standard deviation	• given binomial distribution data, extract the values of n and p in a given context to calculate a z-score, and apply it to the normal curve
• given the values of n and p, calculate a z-score and apply it to the normal curve	
• determine sample spaces for problems involving two or three components	• estimate 95% confidence intervals from data
• given a sample space, draw conclusions about the outcomes of routine problems	• generate a sample space and use it to draw conclusions about the outcomes of non-routine problems
• distinguish between independent and dependent events	• given the probability of one event and the probability of the combined events, determine the probability of the other event
• determine $P(A \text{ and } B)$ for independent events	
• determine $P(A \text{ and } B)$ for dependent events where order is specified	
• determine $P(A \text{ or } B)$ for single events that are mutually exclusive	
• identify complementary events	
• determine the complement to a particular event	
• given the probability of an event, determine the probability of the complement	

- participate in and contribute toward the problem-solving process for problems that require the analysis of statistics and probability studied in Applied Mathematics 30

- complete the solution to problems that require the analysis of statistics and probability studied in Applied Mathematics 30

Examples

Students who achieve the *Acceptable Standard* should be able to answer all the following questions, except for any part of a question labelled $\boxed{\text{SE}}$. Parts labelled $\boxed{\text{SE}}$ are appropriate examples for students who achieve the *Standard of Excellence.*

1. Measure the height of each student in a class, and calculate the mean and standard deviation.

Solution

Answers will vary. Encourage students to become familiar with inputting data into the list function of their calculator and using it to calculate the mean and standard deviation of the data.

2. One of the products of a candy company is a bag of jelly beans that is advertised as containing 200 jelly beans. The machine that fills the bags has been malfunctioning and the workers have been doing manual checks on a frequent basis. The following are counts from 25 bags of jelly beans.

197	200	197	209	204
186	203	201	190	199
204	210	203	211	202
192	189	200	185	200
201	198	188	200	196

a. Calculate the mean and standard deviation of this data.
b. What are the results of having too high / too low of a mean? Explain.
$\boxed{\text{SE}}$ **c.** What is the significance of standard deviation when interpreting the results of the manual checks?

Solution

a. mean = 198.6 jelly beans
$\sigma = 6.92$ jelly beans
b. If the mean is too high, such as 210, then the company would be losing money by overfilling the bags. If the mean is too low, such as 190, then the customers would be losing by not getting what they paid for.
$\boxed{\text{SE}}$ **c.** If the standard of deviation is too high, then people will stop buying the product because they will not be getting a consistent number of jelly beans for their money.

3. The volume of the contents of a soft drink can is normally distributed about a mean of 350 mL, with a standard deviation of 1.5 mL.

a. Calculate the z-score for a can with a content volume of 355 mL.

b. What percentage of cans will have content volumes less than 355 mL?

c. If cans containing less than 346 mL must be rejected, how many cans will be expected to be rejected from every 50 000 cans produced?

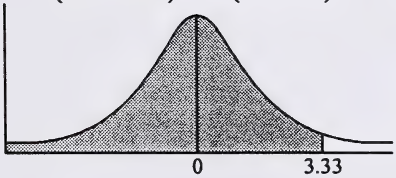
Solution

a.

$$z = \frac{x - \mu}{\sigma} = \frac{355 - 350}{1.5} = 3.33$$

b.

$$P(\text{vol.} < 355) = P(z < 3.33)$$

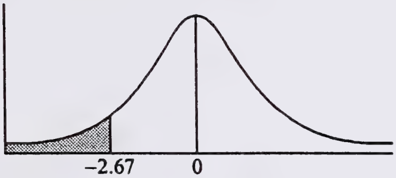


$$P(z < 3.33) = 0.9996$$

Approximately 99.96% will have content volumes less than 355 mL.

c.

$$z = \frac{346 - 350}{1.5} = -\frac{4}{1.5} = -2.67$$



$$P(z \leq -2.67) = 0.0038$$

$$0.0038 \times 50\,000 = 190$$

About 190 cans out of every 50 000 will be rejected.

4. In a particular town, 70% of the students are bused to school. In a random sample of 1 000 students, the mean of the number of students bused to school is expected to be 700, with a standard deviation of 14.49. The probability that in any given sample of 1 000 students, 720 or more students are bused to school is

A. 0.08
B. 0.38
C. 0.62
D. 0.92

Solution

$x = 720$

$n = 700$

$\sigma = 14.49$

$$z = \frac{720 - 700}{14.49}$$

$z = 1.132$

$P(z \geq 1.32) = 1 - 0.9162$

$= 0.0838$

5. Based on past experience, a car salesperson will complete a sale to 10% of the customers that enter the showroom. The salesperson has 200 customers in a particular month,

a. Calculate the mean and standard deviation of this data.

b. Calculate the probability that sales are completed to 10 or fewer customers.

$\boxed{\text{SE}}$ c. Establish a symmetric 95% confidence interval for the number of completed sales.

Solution

a. $\mu = np$ $\quad\quad \sigma = \sqrt{np(1-p)}$

$n = 200$

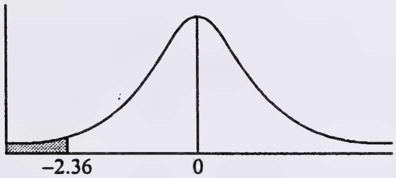
$p = 0.1$

$\mu = 200(0.1)$ $\quad\quad \sigma = \sqrt{200(0.1)(0.9)}$

$= 20$ $\quad\quad = 4.24$

b. $\mu = 20$ $\quad\quad z = \dfrac{10-20}{4.24}$

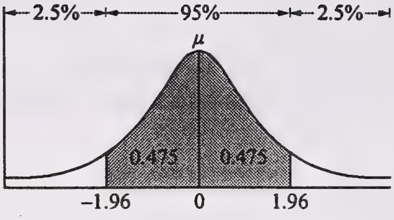
$\sigma = 4.24$ $\quad\quad z = -2.36$



$P(z \le -2.36) = 0.0091$

The probability that the salesperson completes sales to 10 or fewer customers is 0.0091.

SE c.



lower bound ⟵⟶ upper bound

$$z = \frac{x - \mu}{\sigma} \qquad z = \frac{x - \mu}{\sigma}$$

$$-1.96 = \frac{x - 20}{4.24} \qquad 1.96 = \frac{x - 20}{4.24}$$

$$x = 11.69 \qquad x = 28.31$$

We can predict with 95% confidence that the number of completed sales for this salesperson this month will range between 12 and 28.

6. A particular true/false test consists of 40 questions. If a student answers all questions by guessing, what is the probability of the student failing the test by getting 19 or fewer questions correct?

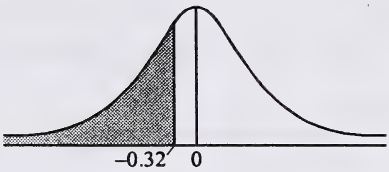
Solution

$$\mu = np \qquad \sigma = \sqrt{np(1-p)} \qquad z = \frac{19-20}{3.16}$$

$$= 40(0.5) \qquad = \sqrt{40(0.5)(0.5)} \qquad = \frac{-1}{3.16}$$

$$\mu = 20 \qquad = \sqrt{10} \qquad z = -0.31645...$$

$$\sigma = 3.16$$



$P(z \leq -0.32) = 0.3745$

The probability of the student failing the test is 0.3745.

7. Determine the probability of rolling a 2 or a 5 on one roll of a 6-sided fair die.

Solution

Sample Space: 1 2 3 4 5 6

Rolling a 2: $\frac{1}{6}$

Rolling a 5: $\frac{1}{6}$

$$P(2 \text{ or } 5) = P(2) + P(5)$$
$$= \frac{1}{6} + \frac{1}{6}$$
$$= \frac{1}{3}$$

8. Construct the sample space for the distribution of boys and girls in a family of 3 children. Calculate the probability that the family consists of 2 boys and 1 girl.

Solution

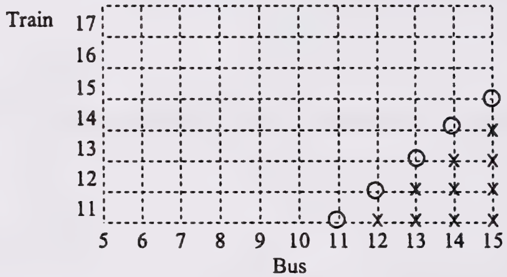
{(G,G,G), (G,G,B), (G,B,G), (G,B,B), (B,G,G), (B,G,B), (B,B,G), (B,B,B)}

$\therefore$ 2 boys and 1 girl $P(2\text{ B and }1\text{G}) = \frac{3}{8} = 0.375$

9. Draw or list the sample space for the following situation. A bus is scheduled to arrive at a train station at any time between 07:05 and 07:15, inclusive. A train is scheduled to arrive between 07:11 and 07:17, inclusive. The arrival of a bus at 07:06 and a train at 07:14 can be represented by the ordered pair (6, 14) on a graph. Times are expressed in whole minutes.

a. How many elements are there in this sample space?
b. How many elements represent the bus and the train arriving at the same time?
c. How many elements represent the bus arriving after the train?
d. What is the probability of the bus arriving after the train?

Solution



a. $11 \times 7 = 77$ points in sample space
b. Five points (circles) represent the bus and train arriving at same time.
c. Ten points (Xs) represent the bus arriving after the train. $(4 + 3 + 2 + 1)$.
d. $P(\text{bus after train}) = \frac{10}{77} = 0.12987$, rounded to 0.13

$\boxed{\text{SE}}$ **10.** In the general population, the IQ scores of individuals are normally distributed with a mean of 110 and a standard deviation of 10. If a large group of people is tested,

a. what proportion will be expected to have IQs between 100 and 120?

b. what is the probability that an individual in the group has an IQ greater than 120?

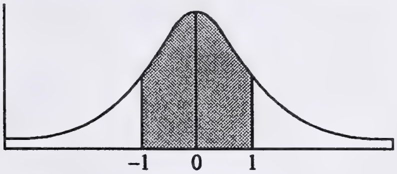
c. what IQ is necessary to be within the top 5% of the sample group?

Solution

a.

$$z = \frac{120 - 110}{10} = 1$$
$$P(s < 120) = 0.8413$$

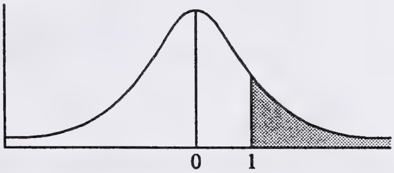
$$z = \frac{100 - 110}{10} = -1$$
$$P(s < 100) = 0.1587$$
$$\therefore P(100 < s < 120) = 0.8413 - 0.1587$$
$$P(100 < s < 120) = 0.6826$$



The proportion of this group with an IQ between 100 and 120 is 0.6826.

b.

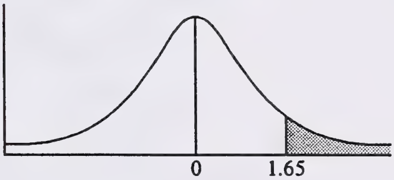
$$P(s < 120) = 0.8413$$
$$P(s > 120) = 1 - 0.8413$$
$$\therefore = 0.1587$$



The probability of a group member having an IQ greater than 120 is 0.1587.

c.

$$P(s < z) = 0.95$$
$$z = 1.65$$
$$1.65 = \frac{x - 110}{10}$$
$$x = 126.5$$



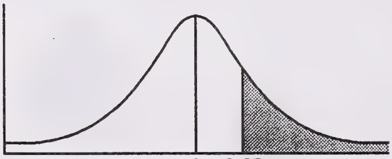
The IQ necessary to be within the top 5% is 126.5.

SE **11.** A sample of 122 people yields a mean body temperature of 36.8°C, with a standard deviation of 0.35°C. Assuming a normal distribution, find the

a. expected number of people with temperatures above 37.0°C
b. expected number of people with temperatures below 36.0°C
c. estimated range of temperatures expected to be contained within the population

Solution

a.



$$z = \frac{37.0 - 36.8}{0.35} = 0.57$$

$$P(t > 37.0) = 1 - P(t < 37.0) = 1 - 0.7157$$

$$P(t > 37.0) = 0.2843$$

If there are 122 people, the expected number of people with a body temperature greater than 37.0° is $(0.2843)(122) = 34.7$, or 34 people.

b.

$$z = \frac{36.0 - 36.8}{0.35} = -2.29$$

$$P(t < 36.0) = 0.0110$$

The expected number of people with a body temperature below 36.0° is $(0.0110)(122) = 1.3$ or 1 person.

c. Maximum temperature should have a z-score of 3.49.

$$3.49 = \frac{x - 36.8}{0.35}$$

$$x = 38.0°\text{C}$$

Minimum temperature should have a z-score of –3.49.

$$-3.49 = \frac{x - 36.8}{0.35}$$

$$x = 35.6°\text{C}$$

Therefore, the range of temperatures is 35.6°C to 38.0°C.

$\boxed{\text{SE}}$ **12.** A multiple-choice test consists of 25 questions, each of which has 4 choices. To pass the test, a student must answer at least 13 questions correctly. What is the probability of a student passing the test by guessing only?

Solution

Since there are 4 choices for each question, the probability of getting a question correct by guessing is $\frac{1}{4}$ or 0.25. The probability of getting it wrong is $\frac{3}{4}$ or 0.75.

$$\begin{aligned} \mu &= np \\ &= 25(0.25) \\ \mu &= 6.25 \\ x &= 13 \end{aligned} \qquad \begin{aligned} \sigma &= \sqrt{np(1-p)} \\ &= \sqrt{25(0.25)(0.75)} \\ \sigma &= 2.165 \end{aligned} \qquad \begin{aligned} z &= \frac{x-\mu}{\sigma} \\ &= \frac{13-6.25}{2.165} \\ z &= 3.12 \end{aligned}$$

$$\begin{aligned} P(x \geq 13) &= P(z \geq 3.12) \\ &= 1 - 0.9991 \\ &= 0.0009 \end{aligned}$$

The probability of passing the test by guessing only is 0.0009.

$\boxed{\text{SE}}$ **13.** The probability that Oleg wins the pole vault is $\frac{1}{6}$. Anya is entered in the 100m race. The probability that they each will win their respective events is $\frac{1}{8}$. What is the probability that Anya will win her race?

$$P(A \text{ and } B) = P(A) \times P((B))$$

$$\frac{1}{8} = \frac{1}{6} \times P(B)$$

$$\left(\frac{1}{8}\right)\left(\frac{1}{6}\right) = P(B)$$

$$\frac{3}{4} = P(B)$$

Design

General Outcomes

Describe and compare everyday phenomena, using either direct or indirect measurement.

Analyze objects, shapes, and processes to solve cost and design problems.

General Notes:

- Students have done considerable work with perimeter, area, scaling, and volume in Applied Mathematics 10 and 20.
- This unit should concentrate on the use of dimensions, unit prices, and estimation to cost projects.
- This unit should include projects wherein a design is given, projects wherein a design is not given, projects wherein a budget is specified, and projects wherein a budget is not specified.
- Generally, the difference between the student at the *Acceptable Standard* and the student at the *Standard of Excellence* is the quality of the solution and the extent to which mathematical processes are demonstrated.
- Outcomes 6.1 and 6.2 are primarily review and are not intended to be the main focus of this unit. They are more of a preparation for outcomes 6.3 and 6.4.
- The objectives for this unit are covered in the project book, not the textbook.

Specific Outcomes

Specific Outcome 6.1
Use dimensions and unit prices to solve problems involving perimeter, area, and volume. [E, PS, V]

Specific Outcome 6.2
Solve problems involving estimation and costing for objects, shapes, or processes when a design is given. [C, E, PS]

Specific Outcome 6.3
Design an object, shape, layout, or process within a specified budget. [PS, C, R, T, V]

6.3 Notes:
- In this outcome, skills from many other areas may be required. Some problems may require various function types on the regression menu of the graphing calculator. It is better to do a few large design projects rather than many small questions.

Specific Outcome 6.4
Use models to estimate the solutions to complex measurement problems. [E, V]

6.4 Note:
- Be aware of the amount of time spent here—general outcome 6 is large because of its explorative nature.

Acceptable Standard	*Standard of Excellence*
The student can	The student can also
• calculate perimeter, surface area, and volume, and can determine costs of composite designs when calculations (i.e., Pythagorean Theorem, trigonometry, solving for any variable in familiar formulas, etc.) are required to find values of necessary dimensions	
• draw sketches and scale diagrams in 2-D from various views when a written description of a design is given	• draw sketches and scale diagrams in 3-D from various views when a written description of a design is given
• solve cost and estimation problems related to perimeter, surface area, and volume consistent with a diagram he or she has drawn	• communicate effectively the meaning from his or her solutions and justify the procedures used
• design objects based on previously learned mathematics, using given information	
• make allowances for changes in given problems	
• describe an estimation process that yields a reasonable result	• design and communicate the most efficient process to yield a desired result
• participate in and contribute toward the problem-solving process for problems that require the analysis of design studied in Applied Mathematics 30	• complete the solution to problems that require the analysis of design studied in Applied Mathematics 30

Examples

Students who achieve the *Acceptable Standard* should be able to answer all the following questions, except for any part of a question labelled $\boxed{\text{SE}}$. Parts labelled $\boxed{\text{SE}}$ are appropriate examples for students who achieve the *Standard of Excellence*.

1. A manufacturer of cylindrical cans uses tin plate that comes in sheets that are 240 cm by 160 cm and cost \$3.20 per sheet. Cans are 6 cm in diameter and 11 cm high, and they have 3 seals each. Seals cost 0.8¢ each to make.

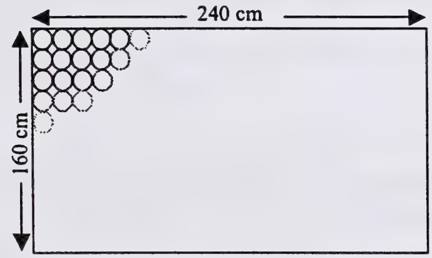
Currently, the manufacturer uses two sheets to make sides and one sheet to make the end pieces.

a. How many ends can be made from one sheet?
b. How many sides can be made from two sheets?
c. Use your answers from part a and part b to determine how many cans can be made from three sheets.
d. How much does it cost to make the cans using this method?
$\boxed{\text{SE}}$ **e.** Design another method for using the three sheets to produce these cans. Is it more or less efficient than the current one? Explain.

Note: This example could be used as a project.

Solution

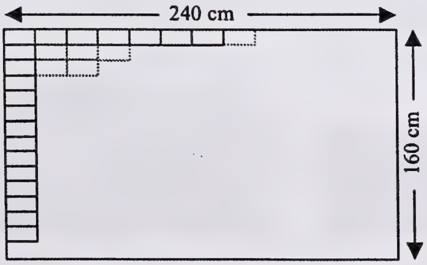
a.



$$\left.\begin{array}{l}\dfrac{240}{6} = 40 \\ \dfrac{160}{6} = 26.\bar{6}\end{array}\right\} 26 \times 40 = 1\,040 \text{ ends}$$

This is enough for 520 cans.

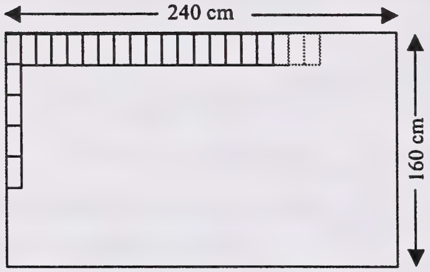
b. Since the cylindrical can has a circumference of 6π and a height of 11 cm, each side can be made using rectangles that are 6π cm $\times$ 11 cm. There are two ways that that these rectangles can be placed on the sheet of tin. The first way is with the longer sides of the rectangles along the longer side of the sheet, as shown below.



With this method, the following number of sides can be made.

$$\left.\begin{array}{l} \dfrac{160}{11} = 14.55 \\[2ex] \dfrac{240}{6\pi} = 12.7 \end{array}\right\} \begin{array}{l} 14 \times 12 = 168 \\ \times 2 \text{ sheets} = 336 \text{ sides can be made.} \end{array}$$

The other way is to put the longer sides of the rectangles along the shorter side of the sheet, as shown below.



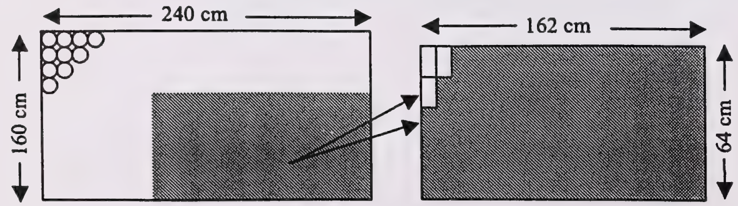
With this method, 336 cans can also be made.

$$\left.\begin{array}{l} \dfrac{160}{6\pi} = 8.49 \\[2ex] \dfrac{240}{11} = 21.8 \end{array}\right\} 8 \times 21 \times 2 = 336$$

c. There are enough ends to make 520 cans but only enough sides to make 336; therefore, 336 cans can be made using this method.

d. $C = \$3.20 \times 3 + 3 \times 336 \times .008$
$= \$9.60 + \$8.06$
$= \$17.66$ to make 336 cans
The cost per can is $0.052 per can.

SE **e.** Since a significant amount of the sheet used for the ends is not used in the above method, part of that sheet could be used to make more sides.



From the unused part of the sheet, you can make an additional 42 sides and an additional 98 ends. This will make a total of 378 sides and a total of 770 ends.

This means you can make a total of 378 cans if you use this method.

The cost for this method is as follows:

Cost for sheets	$\$3.20 \times 3 = \$9.60$
Cost for seals	$\$0.008 \times 3 \times 378 = \$9.07$
Total cost	$18.67
Cost per can	$\dfrac{\$18.67}{378} = \$0.049$

The cost is about the same for this method. However, since this method costs slightly less, over time, the difference in cost may prove significant.

2. A window cleaner has been asked by the management of a large office building to submit a quotation for cleaning the building's windows. The office building has

- 24 floors
- 14 windows per floor, on each side of the building
- 4 sides

To clean the windows, the cleaner starts at the top of one column of windows and works his way to the bottom of that column, cleaning each window on the way down. He then goes back to the top, moves to the second column, and repeats the process. He does this for each side of the building.

From experience, the cleaner knows that it takes

- 120 seconds to clean one window
- 30 seconds to transfer between floors on the same column
- 120 seconds to go back to the top of the building once he has reached the bottom
- 60 seconds to transfer from one column to the next on the same side of the building
- 120 seconds to transfer from one side of the building to the next

a. How many hours will it take to wash all the windows in the building?

b. For every three hours that the window cleaner works, he takes a half-hour break. How many hours will it be before he finishes the entire building?

c. The window washer charges $25 per hour (including the half-hour breaks). He has a base charge of $120, and he wishes to add an additional 10% to the cost of this project so that he can reinvest in his business. Determine the quotation he should give the management of this building.

Solution

a. Determine the time required to complete one side of the building.

First, determine the time required to complete one column of windows.

24 windows @ 120 seconds	= 2 880 seconds
24 floor transfers @ 30 seconds	= 720 seconds
Total	= 3600 seconds or 1 hour

Since there are 14 columns of windows per side, it will take 14 hours to wash the windows on one side.

Add in the time required to transfer from one column to the next.
13 transfers between columns @ 60 seconds = 780 seconds (0.217 hours)

Add in the time required to go back to the top.
14 climbs @ 120 seconds = 1 680 seconds (0.467 hours)

Total time:	14 hours
	0.217 hours
	0.467 hours
Time for one side:	14.684 hours

Determine the time required to complete all four sides.
14.684 hours × 4 sides = 58.736 hours
120 seconds to transfer between sides × 3 = 360 seconds (0.1 hours)

Therefore, it will take 58.836 hours to wash all the windows in the building.

b. $\frac{58.836}{3} = 19.612$ breaks @ 0.5 hours each is 9.806 hours extra

The washer will need $58.836 + 9.806 = 68.642$ hours to complete the project.

c. $120 + 25 \times 68.642 = 1\,836.05$
$1\,836.05 \times 1.10 = 2\,019.66$

The washer should quote the building manager $2 019.66.

3. A compact disc has an outside diameter of 12 cm and an inside diameter of 4.5 cm. The CD can hold 650 MB of information. What is the density of information storage on this CD?

Solution

$A = \pi(6^2 - 2.25^2) = 97.19 \text{ cm}^2$

$\left(\frac{650}{97.19}\right) = 0.669 \text{ MB/cm}^2$

(From Applied Mathematics 30 January 2001 Diploma Examination, Multiple-Choice Question 11)

Use the following information to answer the next question.

In a parkade, cylindrical concrete pillars that will be 0.8 m high and that will have a radius of 0.3 m are to be built. Each pillar will house a rectangular electrical box that measures 0.2 m × 0.2 m × 0.1 m. The pillars will be solid concrete except for this electrical box.

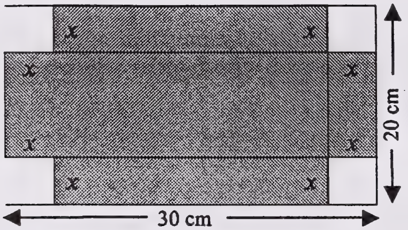
4. If concrete costs $\$134/\text{m}^3$, then the cost of concrete for each pillar will be

A. \$29.77
B. \$30.85
C. \$31.60
D. \$32.70

Solution

$$V = (0.8)(0.3)^2\pi - (0.2 \times 0.2 \times 0.1)$$
$$V = 0.22 \text{ m}^3$$
$$0.22 \text{ m}^3 \times \$134/\text{m}^3 = \$29.77$$

SE 5. A rectangular container is constructed out of a sheet of tin that measures $30 \text{ cm} \times 20 \text{ cm}$. Squares of equivalent sizes are removed from each corner, and then the ends are folded up.



a. Determine an equation that could be used to calculate the volume of the container.
b. Determine an appropriate window setting to graph this function given the context of this problem. Give reasons for your choice.
c. Graph the function in this viewing window and determine the value for x that will give the maximum volume. State the maximum volume.
d. Determine an expression for the surface area of the box.
e. The person using the container wants to maximize its volume while keeping the surface area to a relative minimum. Discuss the values for x that you would choose in order to meet these conditions.

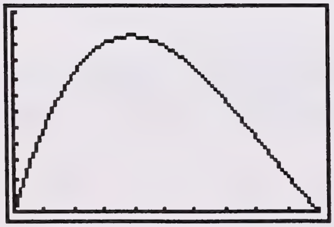
Solution

a. $V = x(20 - 2x)(30 - 2x)$

b. The values for x relate to the length of the sides of the rectangle. Therefore, 0 cm to 10 cm would be a reasonable choice because the length removed must be smaller than 20 cm to result in a container.

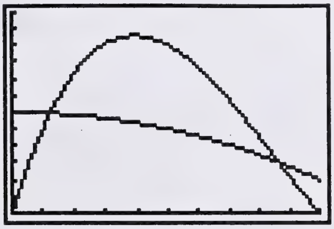
The volume will be greater than 0 cm^3 and must be greater than the surface area. A maximum value of 1 200 seems reasonable.
The viewing window should be x: [0, 10, 1] and y: [0, 1 200, 100].

c.



d. $A = 2x(20 - 2x) + 2x(30 - 2x) + (20 - 2x)(30 - 2x)$

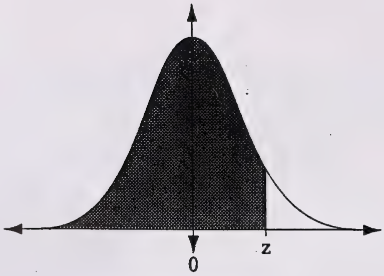
e. If the function for surface area is graphed on the same grid as the volume function, then there is no minimum surface area.



Since the two functions intersect at $x = 1.23$ and $x = 8.62$, and since the maximum volume is between these two values, any value for x that is between 1.23 cm and 8.62 cm would work well. Values less than 1.23 cm result in the volume being too small, as do values greater than 8.62 cm.

Areas Under the Standard Normal Curve

$$z = \frac{x-\mu}{\sigma}$$



z	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00
−3.4	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
−3.3	0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005
−3.2	0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007
−3.1	0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010
−3.0	0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013
−2.9	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019
−2.8	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026
−2.7	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035
−2.6	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047
−2.5	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062
−2.4	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082
−2.3	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107
−2.2	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
−2.1	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179
−2.0	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228
−1.9	0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287
−1.8	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359
−1.7	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446
−1.6	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548
−1.5	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668
−1.4	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
−1.3	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
−1.2	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151
−1.1	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
−1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
−0.9	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841
−0.8	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119
−0.7	0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420
−0.6	0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743
−0.5	0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085
−0.4	0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446
−0.3	0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821
−0.2	0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207
−0.1	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602
−0.0	0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.5	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Applied Mathematics 30 Formula Sheet

Cost and Design

Area

Circle $A = \pi r^2$

Triangle $A = \frac{b \times h}{2}$

Parallelogram $A = b \times h$

Trapezoid $A = h\left(\frac{b_1 + b_2}{2}\right)$

Surface Area

Sphere $SA = 4\pi r^2$

Cylinder $SA = 2\pi r^2 + 2\pi rh$

Cone $SA = \pi r^2 + \pi rs$

Volume

Sphere $V = \frac{4}{3}\pi r^3$

Cylinder $V = \pi r^2 h$

Prism $V = B \cdot h$, where B is the area of the base

Cone $V = \frac{1}{3}\pi r^2 h$

Pyramid $V = \frac{B \cdot h}{3}$, where B is the area of the base

Trigonometry and Vectors

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Statistics and Probability

$$\mu = np$$

$$\sigma = \sqrt{np(1-p)}$$

$$z = \frac{x - \mu}{\sigma}$$

Conf. int: $\mu \pm z\ (\sigma)$

$P(A \text{ or } B) = P(A) + P(B)$

$P(A \text{ and } B) = P(A) \cdot P(B)$

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

Regression Models

$y = a \cdot \sin(bx + c) + d$

$\text{period} = \frac{2\pi}{b}$

$y = ax^2 + bx + c$

$y = ax + b$

$y = a \cdot b^x$

Explanation of Cognitive Levels

Procedures

The assessment of students' knowledge of mathematical procedures should involve recognition, defense, execution, and verification of appropriate procedures and the steps contained within them. Students must appreciate that procedures are created or generated to meet specific needs in an efficient manner, and thus can be modified or extended to fit new situations. The use of technology can allow for conceptual understanding prior to specific skill development. Assessment of students' procedural knowledge will not be limited to an evaluation of their proficiency in performing procedures, but will be extended to reflect the skills presented above.

Certain types of procedural execution can not be tested on diploma examinations because of restrictions in technology. This procedural execution is, however, an integral part of the *Program of Studies* and should be tested in the classroom.

Concepts

An understanding of mathematical concepts goes beyond a mere recall of definitions and recognition of common examples. Assessment of students' knowledge and understanding of mathematical concepts should provide evidence that they can compare, contrast, label, verbalize and define concepts, identify and generate examples and non-examples as well as properties of a given concept, and recognize the various meanings and interpretations of concepts. Students who have developed a conceptual understanding of mathematics can also use models, symbols, and diagrams to represent concepts. Appropriate assessment will also provide evidence of the extent to which students have integrated their knowledge of various concepts.

Problem Solving

Appropriate assessment of problem-solving skills is achieved by allowing students to adapt and extend the mathematics they know, and encouraging the use of strategies to solve unique and unfamiliar problems. Assessment of problem solving involves measuring the extent to which students use these strategies and knowledge, and their ability to verify and interpret results. Students' ability to solve problems develops over time as a result of their experience with relevant situations that present opportunities to solve various types of problems.

"Problem solving can be used effectively as a context in which students will learn new concepts and skills." (Hiebert, 1998).

Evidence of problem solving skills is often linked to clarity of communication. Students demonstrating strong problem-solving skills should be able to clearly explain the process they have chosen using clear language and appropriate mathematical notation and conventions.

Calculator Active Questions

"Improvements in technology, and its increased availability in schools, have changed the focus of mathematics education. The time saved by using calculators and computers to perform complex calculations can be used to help students better understand mathematical concepts. Students can then understand the relationships among concepts and use these relationships to solve problems." (*Applied and Pure Mathematics Program of Studies*, page 9).

It is expected that students writing an Applied Mathematics 30 diploma examination will have an approved graphing calculator and be proficient in its use (see *Calculator Policy* for a list of approved calculators). If students are asked to provide a graph in their solution, they will be required to provide the dimensions of the graphing window used. The dimensions are to be communicated in the following manner.

x: $[x_{min}, x_{max}, x_{scl}]$ y: $[y_{min}, y_{max}, y_{scl}]$

For example, the window format: x: [–3, 5, 1] y: [–50, 50, 10] would mean the x-axis is scaled from –3 to 5 in increments of 1 and the y-axis is scaled from –50 to 50 in increments of 10.

Some questions will require the use of a graphing calculator.

E.g., the following table shows the average daily temperatures in Winnipeg, Manitoba, by month over a period of one year.

Average Daily Temperature in Winnipeg, Manitoba	
Month	**Average Temperature**
January	–18
February	–18
March	–11
April	0
May	11
June	18
July	20
August	19
September	14
October	7
November	–8
December	–17

a. Use a graphing calculator to plot the data.

b. Determine an approximate sinusoidal function for this data. Express the values to the nearest hundredth. Graph this function in the viewing window provided and state the window settings.

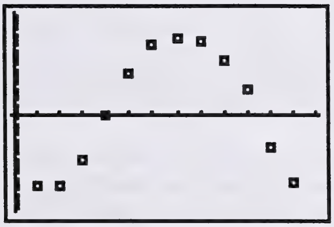
x: [, ,]

y: [, ,]

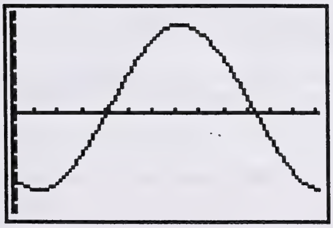
function: ______________________________

Solution:

a. A possible solution:



b.



x: $\left[\ 0,\ 13,\ 1\ \right]$

y: $\left[\ -25,\ 25,\ 1\ \right]$

function: $\underline{y = 20.61\sin(0.52x - 2.16) + 1.21}$

Alberta Learning Diploma Examinations Program Calculator Policy for Examinations Written Beginning in the School Year 2000–01

Rationale

The new Alberta Learning *Program of Studies* for mathematics began implementation with Grade 10 Mathematics in September 1998. It encourages teachers to integrate the use of technology into their lessons. This use of technology will assist teachers in emphasizing learning activities that develop high-level thinking skills and that allow students to solve complex, multifaceted problems. The *Mathematics Applied and Pure Program of Studies* states:

> *Improvements in technology, and its increased availability in schools, have changed the focus of mathematics education. The time saved by using calculators or computers to perform complex calculations can be used to help students better understand mathematical concepts. Students can then understand the relationships among concepts and use these relationships to solve problems.*
>
> *Calculators and computers can be used as tools to*
> - *explore and demonstrate mathematical relationships and patterns*
> - *organize and display data*
> - *assist with solving problems*
> - *decrease the time spent on tedious computations*
> - *simulate situations*

Professional mathematics organizations such as the National Council of Teachers of Mathematics (NCTM) and the Mathematics Council, Alberta Teachers' Association (MCATA), have strongly endorsed the use of graphing calculators in mathematical instruction. They also emphasize that if calculator and computer technologies are now accepted as part of the environment in which students learn and do mathematics, such tools should also be available to students in most assessment situations.

As a result, the calculator policy for the Pure Mathematics 30 and Applied Mathematics 30 diploma examinations, includes the use of graphing calculators. Although many questions will not require the use of a graphing calculator, there will be some questions for which the use of a graphing calculator will be required.

All science diploma examinations and examinations in Mathematics 30 (old program) and Mathematics 33 will require the use of a scientific calculator or of a graphing calculator approved by Alberta Learning.

Note: This policy applies to Alberta Learning diploma examinations and field tests only. School calculator policies may differ depending on grade level or topic studied.

Definition

The calculator must be a hand-held device designed primarily for mathematical computations, including logarithmic and trigonometric functions, as well as for graphing functions. Included in this definition are those scientific calculators having graphing and programmable features.

Policy

To ensure compatibility with the provincial *Programs of Study* and equity and fairness to all students, Alberta Learning expects students to use calculators, as defined, when writing diploma examinations in mathematics and the sciences. **Pure Mathematics 30 and Applied Mathematics 30 require the use of an approved graphing calculator for diploma examinations. All science diploma examinations and examinations in Mathematics 30 (Old Program) and Mathematics 33 require the use of a scientific calculator or an approved graphing calculator.**

Procedures

1. At the beginning of **any** mathematics or science diploma examination course, teachers must advise students of the types of calculators approved by Alberta Learning for use when writing diploma examinations in these courses.
2. Students must clear **all** programmable calculators, both graphing and scientific, that are brought into diploma examinations of all information that is stored in the programmable or parametric memory.
3. Students must not bring external devices (peripherals) to support calculators into any examination. Such devices include manuals, printed or electronic cards, printers, memory expansion chips or cards, external keyboards, CD-ROMs, libraries, or any annotations that outline operational procedures.
4. In preparation for calculator failure, students may bring extra batteries and/or approved calculators into the examination room.
5. During examinations, supervising teachers must ensure that
   - calculators operate in silent mode
   - students do not share calculators or information contained within them
   - calculator cases are not available to students
   - programmable calculator memories, including parametric memories, have been cleared
   - only graphing calculators on the current list approved by Alberta Learning are used

Calculator Criteria

The following criteria will be used to select acceptable calculators.

Minimum calculator properties required

1. Function graphing capabilities with display
   —includes displaying more than one function on the screen at a time, tracing a function
2. Standard scientific calculator operations
   —e.g., sine, cosine, tangent, inverse functions, logarithms, power (x^n)
3. Statistical functions in 1 and 2 variables
   —mean, median, mode, standard deviation, bivariate data, regression models
4. List capabilities
5. Matrix capabilities
   —scalar multiplication, addition, and subtraction

Unacceptable calculator properties during examinations

1. Built-in notes (definitions or explanations in alpha notation), e.g., libraries
2. Upgrades that include built-in notes
3. Remote communication ability

The following list of approved calculators is provided to assist students and teachers in the selection of graphing calculators that conform to the requirements stated in the definition and to the stated criteria. The list will be updated annually.

Note: All the calculators listed below meet the "required properties". They do not have any "unacceptable properties" and so can be used on the mathematics and science diploma examinations. However, students and teachers should recognize that the different models of calculators listed have a range of capabilities, and the choice of which model to use or purchase will require personal or teacher analysis of the machines' capabilities and one's individual or school circumstances.

The List of Approved Graphing Calculators

Brands	*Casio*	*Sharp*	*Hewlett-Packard*	*Texas Instruments*
Models	Algebra FX 2.0	El-9600C	HP39G*	TI-82* TI-83 TI-83 Plus TI-86 TI-89 TI-92 Plus

*** The TI-82 calculator will remain on the approved list for the 2001–2002 school year and will then be deleted from the approved list. The HP39G calculator will remain on the approved list for the 2001–2002 and 2002–2003 school years and then will be deleted from the approved list. These calculators are not recommended for students entering high school mathematics who wish to use a calculator throughout their high school program.**

The following calculators meet the graphing calculator criteria and are approved, but are no longer commercially manufactured.

Brands	*Casio*	*Sharp*	*Texas Instruments*
Models	FX-9700 series CFX-9800G CFX-9850G CFX-9850GA Plus	El-9600 El-9200 El-9300C	TI-92

Note: Instructions for clearing calculator memories are posted in the Alberta Learning web site: *http://www.learning.gov.ab.ca/K_12/testing/diploma/bulletins/essential/clearing_calc.asp*

For Further Information

If you have any questions or comments about this policy, please contact the Assistant Director of the Mathematics/Science Diploma Examination Unit, Learner Assessment Branch, at (780)427-0010, fax (780)422-4454. To call toll-free from outside of Edmonton, dial 310-0000.

Keystrokes Required For Clearing **Approved** Calculators

The following information is provided to help familiarize students, teachers, and presiding examiners with the procedures involved in clearing all calculator memories.

Procedures to be followed prior to the writing of a diploma examination.

1. At the beginning of **any** mathematics or science diploma examination course, teachers must advise students of the types of calculators approved by Alberta Learning for use when writing diploma examinations.
2. Students must clear **all** programmable calculators, both graphing and scientific, that are brought into diploma examinations. All information that is stored in the programmable or parametric memory must be cleared.
3. Presiding examiners are responsible for ensuring that
    a. all calculators operate in silent mode
    b. students do not share calculators or information contained within them
    c. calculator cases are stored on the floor throughout the examination
    d. all examination rules are followed

Note 1
If you find that there are problems with any of the clearing techniques, please contact the Assistant Director of the Mathematics/Science Diploma Examination Unit, Learner Assessment Branch, at (780)427-0010, fax (780)422-4454. To call toll-free from outside of Edmonton, dial 310-0000.

Note 2
Resetting calculators may result in altering the calculator mode settings. Please remember to check the mode settings before proceeding with the diploma examination.

Note 3
Programs downloaded from the web are not allowed on the calculators used during diploma examinations and will be erased by these procedures.

Note 4
The memory values given on the next pages refer to memory expected to be available as a factory setting. The values available in student calculators should match these values when the calculator has been reset. If the values in the student calculators do not match these values then the calculators should be reset a second time. If this fails to change the values, then the calculator should not be used on the examination.

Texas Instruments	**Memory remaining**
TI 82 2^{nd} + (MEM) 3 (Reset) 2 (Reset) **Note: If, on clearing, the screen is blank, the contrast needs to be reset. To do this, use 2^{nd} ⇑ both repeatedly.**	**2^{nd} + MEM FREE 28734** **1**
TI 83 2^{nd} + (MEM) 5 (Reset) 1 (All memory) 2 (Reset) **Note: If, on clearing, the screen is blank, the contrast needs to be reset. To do this, use 2^{nd} ⇑ both repeatedly.**	**2^{nd} + RAM 27118** **1**
TI 83 Plus 2^{nd} + (MEM) 7 (Reset) ➤➤ (All) Enter 2 (Reset) Note: **If, on clearing, the screen is blank, the contrast needs to be reset. To do this, use 2^{nd} ⇑ both repeatedly.**	**2^{nd} + RAM 24317** **2 ARC 163840**
TI 86 2^{nd} 3 (MEM menu) F3 (Reset) F1 (All) F4 (Yes) **Note: If, on clearing, the screen is blank, the contrast needs to be reset. To do this, use 2^{nd} ⇑ both repeatedly.**	**2^{nd} 3 MEM FREE 98226** **F1**
TI 89 2^{nd} 6 (MEM) F1 (All) 1 (Reset) Enter	**2^{nd} 6 RAM 199154** **ARC 393204**
TI 92 2^{nd} 6 (MEM) F1 (Reset) 1 (All) Enter **Note: If, on clearing, the screen is blank, the contrast needs to be reset. To do this, use ◊ (green) and + or – repeatedly.**	**2^{nd} 6 System 61064** **Memory Free 70008**

Casio

Memory remaining

Casio FX9700 series Go to **Menu** Cursor to **Reset** **EXE** (All Memory) **F1** (Yes – Reset All)	**Menu** **24 000 Bytes available** **EXE** **Clear** **Shift** **MDISP** (CAPA)
Casio CFX9800G Go to **Menu** **Options** **EXE** (Memory) **Reset** **EXE** (Reset) **F1** (Yes – Reset All)	**Menu** **24 000 Bytes available** **EXE** **Clear** **Shift** **MDISP** (CAPA)
Casio CFX-9850G, CFX-9850GA, CFX-9850GA Plus Go to **Menu** Cursor to **Mem** **EXE** (Memory) ⇓ (Reset) **EXE** **F1** (Yes – Reset All) Note: For the GA plus, use Alpha F.	**Menu** **30677** **Alpha E** **EXE** **Check usage**
Casio Algebra FX 2.0 Go to **Menu** Cursor to **SYSTEM** **EXE** System manager **F5** Reset **F2** Reset (Clear) Main Memories? **EXE** Yes – Main Memories Cleared **ESC** Reset **F3** Reset (Clear) Storage Memories? **EXE** Test – Storage Memories Cleared **Menu** **Note: A tutorial system exists on this calculator and may be locked out for 180 minutes by following steps illustrated in the manual. Since the information in the tutorial has no effect upon the Pure 30/Applied 30 Mathematics topics, application of this process is not necessary. This may have implications for Pure 10-20/Applied 10-20topics.**	Go to **Menu** Cursor to **Memory** **EXE** **Memory Manager** **F1** **Current Area No Program (146912 Bytes Free.)** **F6** **Storage Area No Program (786240 Bytes Free.)** **Menu**

Sharp

Memory remaining

Sharp EL 9600 and 9600 C 2^{nd} **XθTN** (Option) **Log** (Reset) **2** (All Memory) **CL** (Clear all data) **Note: There is also a reset switch on the back. (Use round tip of pen, press, then CL)**	**2^{nd} XθTN 18562** ⇓

Hewlett-Packard

Memory Remaining

Hewlett-Packard HP39G Hold down simultaneously: **ON** Menu Key 1 Menu Key 6 Release **OK** (Menu Key 6)	**SHIFT › Memory Manager 234 K** **Note:** All items in this window should read 0 KB.

Directing Words

Discuss

The word "discuss" will not be used as a directing word on math and science diploma examinations because it is not used consistently to mean a single activity.

The following words are specific in meaning.

Algebraically

Using mathematical procedures

Compare

Show the character or relative values of two things by pointing out their *similarities* and *differences*

Conclude

State a logical end based on reasoning and/or evidence

Contrast/Distinguish

Point out the *differences* between two things that have similar or comparable natures

Define

Provide the essential qualities or meaning of a word or concept; make distinct and clear by marking out the limits

Describe

Give a written account or represent the characteristics of something by a figure, model, or picture

Design/Plan

Construct a plan, i.e, a detailed sequence of actions, for a specific purpose

Determine

Find a solution, to a specified degree of accuracy, to a problem by showing appropriate formulas, procedures, and calculations

Evaluate

Give the significance or worth of something by identifying the good and bad points or the advantages and disadvantages

Explain

Make clear what is not immediately obvious or entirely known; give the cause of or reason for; make known in detail

Graphically

Using a drawing that is produced electronically or by hand, and that shows a relation between certain sets of numbers

How

Show in what manner or way, with what meaning

Hypothesize
Form a tentative proposition intended as a possible explanation for an observed phenomenon; i.e., a possible cause for a specific effect. The proposition should be testable logically and/or empirically.

Identify
Recognize and select as having the characteristics of something

Illustrate
Make clear by giving an example. The form of the example must be specified in the question; i.e., word description, sketch, or diagram.

Infer
Form a generalization from sample data; arrive at a conclusion by reasoning from evidence

Interpret
Tell the meaning of something; present information in a new form that adds meaning to the original data

Justify/Show How
Show reasons for or give facts that support a position

Model
Find a model (in mathematics, a model of a situation is a pattern that is supposed to represent or set a standard for a real situation) that does a good job of representing a situation

Outline
Give, in an organized fashion, the essential parts of something. The form of the outline must be specified in the question; i.e., lists, flow charts, concept maps.

Predict
Tell in advance on the basis of empirical evidence and/or logic

Prove
Establish the truth of validity of a statement for the general case by giving factual evidence or logical argument

Relate
Show logical or causal connection between things

Solve
Give a solution for a problem; i.e., explanation in words and/or numbers

Summarize
Give a brief account of the main points

Trace
Give a step-by-step description of the development

Verify
Establish, by substitution for a particular case or by geometric comparison, the truth of a statement

Why
Show the cause, reason, or purpose